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Experimental Evaluation of Packed Bed Heat Transfer Relations

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Abstract. The objective of this research is to obtain a relationship between Nusselt and Reynolds numbers of fluid flow in packed bed containing spheres of different sizes. Also, to develop a universal relationship for the Nusselt number in a wide range of Reynolds number and compare the results with those of empty tube. The experimental work covered a range of Reynolds numbers, based on tube diameter, from 2167 to 19400 at various spherical P.V.C. porous media particle diameters from 0.003 to 0.012 m as well as without porous media (empty section). The results of this experimental work show that the Nusselt number improves by a factor of 3.8 to 6.3 times when using porous media for sphere diameters ranging from 0.012 to 0.003 m respectively as compared to the empty tube. Small size spherical porous media yield much better results than large sizes because they offer more turbulence and eddy, while higher pressure drop develops. These results improve the heat transfer coefficient.

Keywords: Packed bed, Heat transfer.

List of Symbols and Abbreviations

A   Area, m^2
a   Constant
b   Constant
C   Constant
C_p Specific heat at constant pressure, kJ/kg.K
D   Inner tube diameter, m
d   Sphere diameter, m
f   Constant
h   Convective heat transfer coefficient, $W/m^2.K$
k   Thermal conductivity, W/m.K
L   Length, m
Nu_D Nusselt number based on tube diameter = h.D / k
$Nu_{w.o}$ Nusselt number based on tube diameter when using empty tube
Pr   Prandtl number = $\mu_f . C_p / k$
ΔP Pressure drop across the test section, Pa
Q   Rate of heat transfer, Watt
Re_D Reynolds Number based on tube diameter $= \rho.\mathbf{v}.D / \mu_f$
t_{hi} Hot water inlet temperature, ºC
t_{ho} Hot water outlet temperature, ºC
t_m Mean surface temperature, ºC
V   Hot water flow rate, m^3/sec
V_t Inside volume of steel tube when filled with water, m^3
V_S Spheres volume, m^3
v   Hot water velocity, m/sec

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Greek Letters

ε Porosity of the bed
ρ Density, kg/ m^3
μ_f Viscosity, kg/m s

Subscripts

f Fluid
hi Hot at inlet state
ho Hot at exit state
m Mean for the fluid
s Surface, Solid
t Tube

1. Introduction

Porous media are utilized in a wide range of engineering applications such as chemical catalytic reactors, compact thermal collectors, storage systems, heat pipe technology, building thermal insulation, combustion, solid matrix heat exchangers, petroleum reservoirs, geothermal operations, packed spheres ground water hydrology and the manufacturing of numerous products in chemical industry.

Jonsson and Catton [1] have studied the effect of Prandtl number of a medium on heat transfer across a horizontal layer. They found that the heat transfer coefficient increases with smaller particles. Nasr *et al.* [2] have studied the combination of conduction and radiation heat transfer in packed beds. They found that higher effective thermal conductivities were obtained with larger particles and higher thermal conductivity packing materials. Wu and Hwang [3] have studied experimentally and theoretically the fluid flow and heat transfer characteristics inside packed beds. They found that the heat transfer coefficient is greatly affected by Reynolds number and porosity. Elkady [4] has experimentally simulated the forced convection heat transfer from a circular pipe filled with porous media. He found that the average Nusselt number increases with increasing particle to pipe diameter ratio and Reynolds number. Afify and Berbish [5] have studied non-Darcian forced convection heat transfer and pressure drop in a circular tube filled with a packed bed. They found that the local Nusselt number is increased with decreasing the sphere diameter at constant Reynolds number. Poulikakos and Renken [6] have studied the forced convection in a channel filled with a fluid saturated porous medium. They found that increasing the sphere diameter to channel half width ratio yields an overall increase in fluid velocity and heat transfer from the fluid to wall. Vafai and Kim [7] have studied the fully developed forced convection in a porous channel bounded by parallel plates. They found that the variation of the Nusselt number for fully developed temperature and velocity fields is a function of the Darcy number only. Vafai and Sozen [8] have studied the forced convective flow of a gas through a packed bed. They found that the local thermal equilibrium assumption should not be carried out for high (Re) and/or high (Da) flows in a packed bed. Vafai and Kim [9] have studied the convective flow and heat transfer through a composite porous consists of a fluid layer overlaying a porous substrate. They found that the Darcy number is directly related to the permeability of the porous medium. Breton, Caltagirone and Arquis [10] have studied natural convection in a square cavity in which differentially heated vertical walls are covered with thin porous layers. They found that the reduction of overall Nusselt number increases with Rayleigh number. Haji [11] has studied the fully developed heat transfer to a fluid flow in a rectangular passages filled with porous materials.

2. Experimental Apparatus and Measuring Instruments

A schematic diagram of the experimental set up is shown in Fig. 1. The main parts of the experimental apparatus are test section, hot water system, cooling water system, and measuring instruments.

2.1. Test section

The test section contains four parts: test tube, outer tube, entrance and exit tubes.

2.1.1. Test tube

The test tube, which is made of steel contains the porous media and allows hot water to flow through it. It has 0.0425 m inner diameter, 0.048 m outer diameter and 0.5 m length. The thermocouples are distributed on four sections along the test tube surface, as shown in Fig. 2. The distance between each section is equal to 0.1 m. The temperature of each section is measured by means of three thermocouples distributed through the section. The 1st one at the top point of the section, the second one is 90 degree from the first while the 3 rd one located at the bottom point in the section.

2.1.2. Outer tube

The outer tube is made from P.V.C. tube, which allows the cold water to pass through the jacket between the test tube and the outer tube, in order to achieve the isothermal surface through the test section.

No.	Title
1	Overhead tank
2	Electric heater
3	Control valve
4	Entrance tube
5	Test section
6	Exit tube
7	Lower tank
8	Lift pump
9	U- Tube manometer
10	Thermocouple
11	Hose

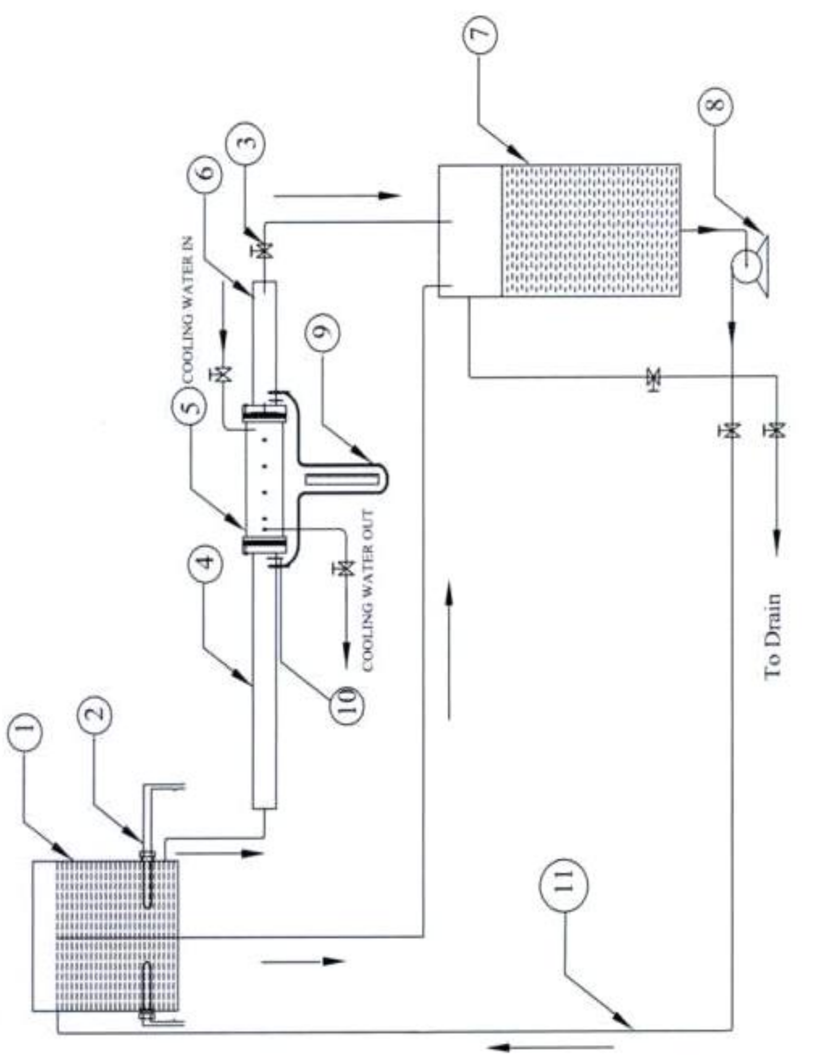

Fig. (1). Schematic diagram of the experimental set up.

2.1.3. Entrance and exit tubes

The design of the entrance tube is made to guarantee fully developed flow and almost uniform velocity at the inlet of the test section. The upstream and downstream parts of the steel tube are made from P.V.C. tube and allow thermocouples to measure the water temperatures before and after the test section.

The entrance length for the test section is designed to obtain almost uniform velocity upstream of the test section, which requires a length around 20 times the diameter as presented in [12, 13]. Therefore, the P.V.C. tube upstream the test section is selected with 1 m length, 0.0425 m inner diameter and 0.048 m outer diameter. The exit P.V.C. tube has an inner diameter of 0.0425 m, an outer diameter of 0.048 m and a length of 0.5 m. The pressure drop across the test section was measured by U-tube manometer connected upstream and downstream of the test section by using two copper taps.

2.2. Hot water system

The hot water system consists of an overhead tank, a lower collecting tank, water pump, valves and hoses. The overhead tank is used to obtain a constant water head of the hot water flow. To ensure the steadiness of the flow, the head in the tank is kept constant by means of an adjustable overflow pipe. The water is heated by two electric heaters (2 kW each) which are submerged in the overhead tank. The lower tank is used to collect the water for recycling. A water pump is used to lift the water from the lower tank to the overhead tank. Valves are used to control the hot water flow rate and to purge the air from the test rig.

2.3. Cooling water system

The open loop cold water system is obtained by cold water supply, and controlled by adjusting the control valve.

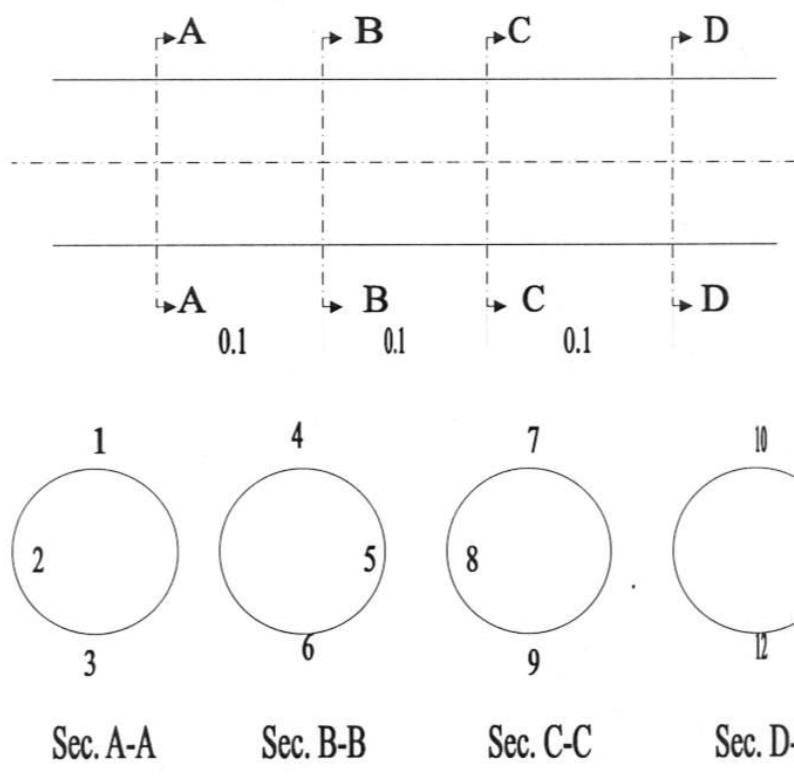

Fig. (2). Thermocouples distribution along the test section.

2.4. Measuring instruments

Measuring instruments are used to measure the physical quantities required to calculate the heat transfer coefficient. The measuring instruments are: thermocouples, a U-tube manometer, digital thermometers, a digital stop watch and a graduated vessel.

2.5. Porous media

The porous media used for the runs is P.V.C. spheres with diameters (0.003, 0.004, 0.005, 0.006, 0.008 and 0.012 m). Void fraction during this work is measured experimentally. The following equations are used to evaluate the void fraction.

$$\varepsilon = \left(\frac{V_t - V_S}{V_t} \right) \tag{1}$$

The relationship between packed bed diameter ratio and void fraction is represented in Table 1.

Table (1). Relationship between packed bed diameter ratio and void fraction.

d *10^3 m	D/d	V_t *10^6 m^3	($V_t - V_S$)*10^6 m^3	ε
3	14.17	709.31	280	0.386
4	10.63	709.31	289	0.396
5	8.50	709.31	294	0.403
6	7.08	709.31	298	0.408
8	5.31	709.31	302	0.414
12	3.54	709.31	308	0.421

3. Method of Calculation

Main assumptions for calculations are:

- The value of Re_D is calculated based on the velocity across porous media and calculated by the following procedures:

$$\varepsilon = (V_t - V_S) / V_t = V_f / V_t \tag{2}$$

$$\varepsilon = V_f / V_t = A_f L_f / A_t L_t \tag{3}$$

If $L_f = L_t$ and $A_f / A_t = (D_f / D)^2$, then:

$$\varepsilon = A_f / A_t \tag{4}$$

$$v_f . A_f = v_t . A_t \tag{5}$$

$$v_f = v_t (A_t / A_f) \tag{6}$$

$$v_f = v_t / \varepsilon \qquad \text{(velocity across porous media)} \tag{7}$$

$$Re = (\rho_f v_t D_t)/\mu_f \qquad \text{(Entrance tube)} \tag{8}$$

$$Re_D = (\rho_f v_f D_f)/\mu_f \qquad \text{(across porous media)} \tag{9}$$

$$Re_D / Re = v_f D_f / v_t D_t = 1/(\varepsilon)^{0.5} \tag{10}$$

$$Re_D = Re /(\varepsilon)^{0.5} \tag{11}$$

The following procedure is followed to calculate Nusselt number:

$$Q = \rho_f V c_p (t_{hi}-t_{ho}) \qquad \text{Watt} \tag{12}$$

$$h = \frac{Q}{(\pi DL(t_m - t_S))} \qquad \text{W/m}^2\text{.K} \tag{13}$$

where:

$$t_s = \frac{t_1 + 2*t_2 + t_3 + t_4 + 2*t_5 + t_6 + t_7 + 2*t_8 + t_9 + t_{10} + 2*t_{11} + t_{12}}{16}$$

$$t_m = \frac{t_{hi} + t_{ho}}{2}$$

$$Nu_D = h\, D/ k_f \tag{14}$$

4. Results and Discussions

The experimental work covers a wide range of Reynolds number starting from 2167 to 19400, and various spherical P.V.C. porous media diameters from 0.003 to 0.012 m. Also, experimental work covers the empty tube. Uncertainty in the measured Nusselt number is estimated as 3.9%, while it is 1.7 % in Reynolds number.

4.1. Effect of diameter ratio on heat transfer

Figures 3 and 4 show the relationship between the heat transfer coefficient, Nusselt number and Reynolds number with and without using porous media.

The effect of diameter ratio is shown. The large diameter ratio is the best case compared with the low diameter ratio because the large diameter ratio has low porosity compared with the small diameter ratio. Therefore, more turbulence occurs generating more eddies resulting in improving the Nusselt number.

The experimental work was carried out to show the effects of porous media, the results reveal that the heat transfer coefficient without using porous media is lower than that with porous media due to less turbulence, less eddies and approximately zero pressure drop on the test section.

Figure 5 shows the relationship between the Nusselt number and the diameter ratio for various values of Reynolds number. Note that the Nusselt number increases with increasing the diameter ratio. Also, the slopes of the curves are increased with increasing the Reynolds number.

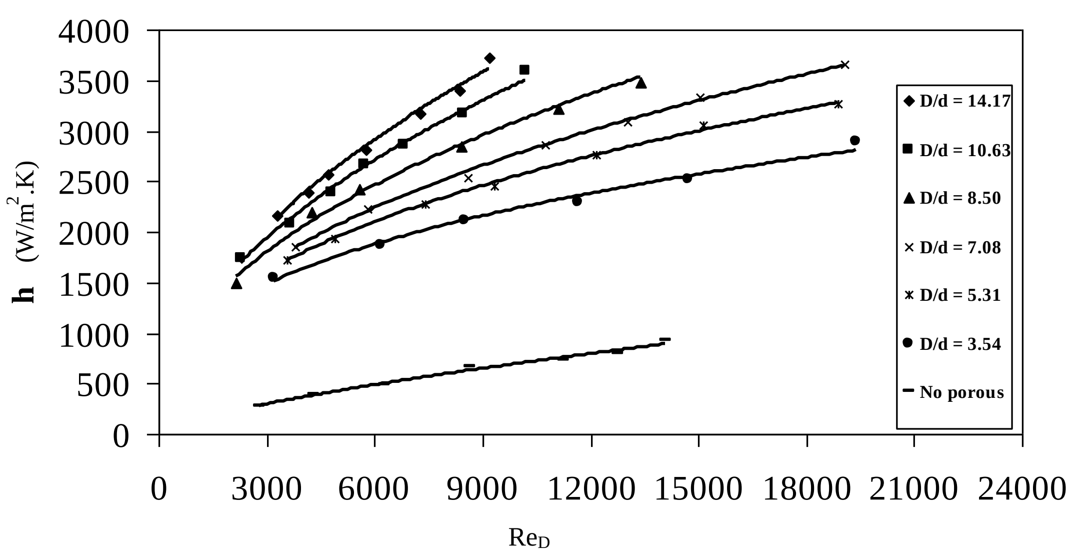

Fig. (3). Variation of heat transfer coefficients with Renolds numbers with and without porous media at the same conditions.

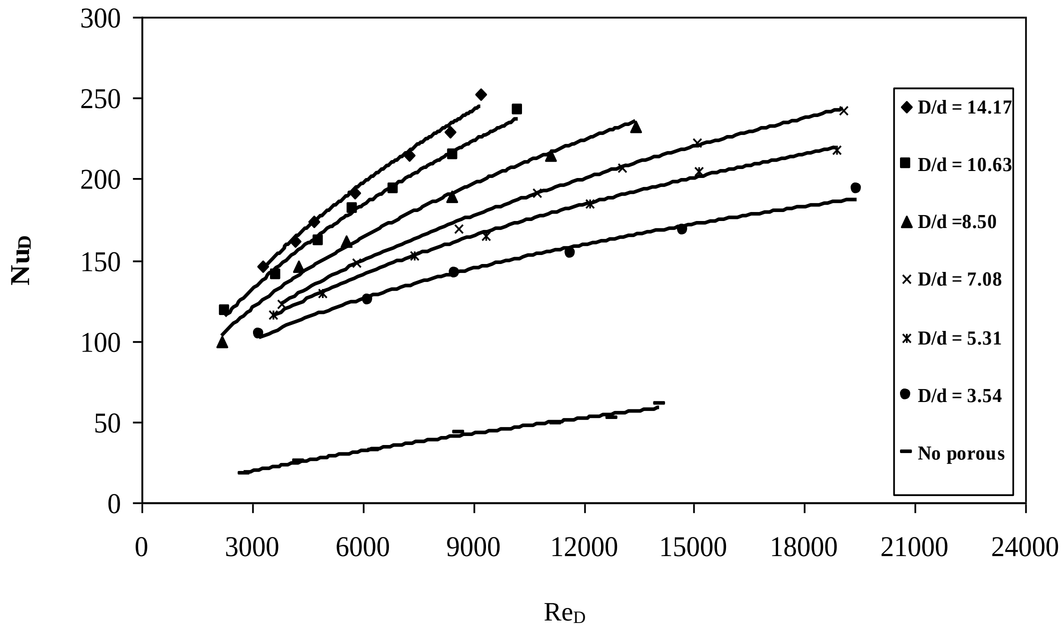

Fig. (4). Variation of Nusselt numbers with Reynolds numbers with and without porous media at the same conditions.

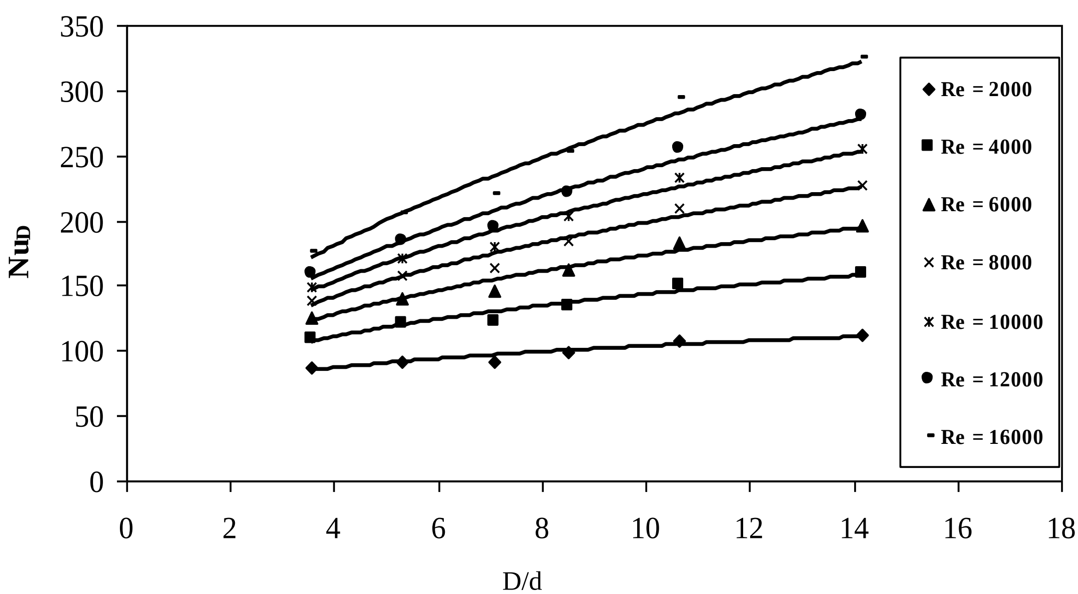

Fig. (5). Variation of Nusselt numbers with Diameter ratio for P.V.C. spheres for different Reynolds numbers at the same conditions.

4.2. Correlations of heat transfer results

The correlation between the Nusselt number and the Reynolds number at various porous sphere diameters is shown in Fig. 4, and can be addressed in the following general form:

$$Nu_D = a_1 \,\mathrm{Re}_D{}^{b_1} \tag{15}$$

Referring to these results, the values of the coefficients (a_1) and (b_1) are a function of diameter ratio.

$$a_1 = c_1\left(\frac{D}{d}\right)^{f_1} \text{ and } b_1 = c_2\left(\frac{D}{d}\right)^{f_2}$$

The final relationship becomes:

$$Nu_D = 17.30\left(\frac{D}{d}\right)^{-0.77} \mathrm{Re}_D{}^{0.235\left(\frac{D}{d}\right)^{0.3}} \tag{16}$$

For: $2167 \le \mathrm{Re}_D \le 19400$ and

$3.54 \le D/d \le 14.16$.

This relation gives error in the range ± 7%. The relationship is correlated as shown in Fig. 6. For empty tube the relationship between the Nusselt number and Reynolds number without using porous media is:

$$Nu_D = 0.042\,\mathrm{Re}_D{}^{0.76} \tag{17}$$

For: $2778 \le \mathrm{Re}_D \le 14000$ and

$1.5 \le \mathrm{Pr} \le 3.4$

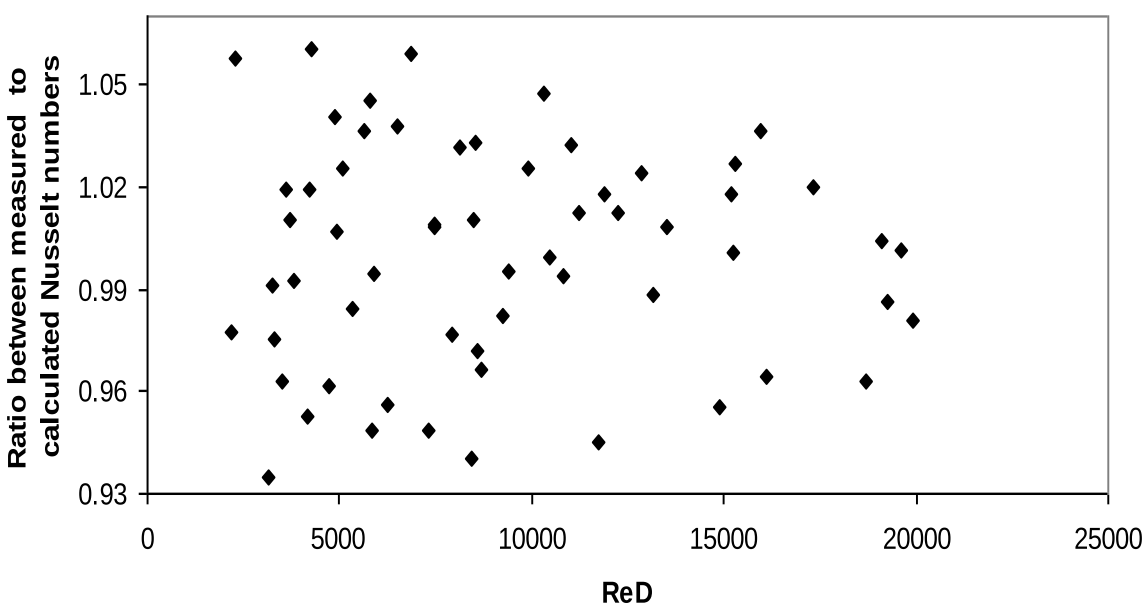

Fig. (6). Variation of the ratio of measured to calculated Nusselt numbers with Reynolds numbers.

4.3. Effect of packed bed on heat transfer enhancement

The results of this experimental work, which is shown by Eqs. (16) and (17) prove that the Nusselt number and heat transfer coefficient are improved due to the use of porous media as shown in Table 2.

The results of this experimental work show that the Nusselt number improves by a factor of 3.8 to 6.3 times when using porous media for porous diameters from 0.012 to 0.003 m respectively as compared to the empty tube.

Table (2). Ratio between packed bed Nusselt number and empty tube Nusselt number for different porous media diameters.

d *10^3 m	$Nu_D / Nu_{W.O}$ Re_D 3000	$Nu_D / Nu_{W.O}$ Re_D 6000	$Nu_D / Nu_{W.O}$ Re_D 9000	$Nu_D / Nu_{W.O}$ Re_D 12000	$Nu_D / Nu_{W.O}$ Re_D 15000	$\left(\frac{Nu_D}{Nu_{W.O}}\right)_{av}$
3	7.8	6.6	6.0	5.6	5.3	6.3
4	6.9	5.7	5.1	4.7	4.4	5.4
5	6.4	5.2	4.6	4.2	3.9	4.9
6	6.1	4.8	4.2	3.8	3.5	4.5
8	5.8	4.5	3.8	3.5	3.2	4.1
12	5.5	4.1	3.5	3.1	2.8	3.8

4.4. Effect of diameter ratio on pressure drop

Figure 7 shows the relationship between pressure drop across the test section and Reynolds number for various sphere diameters. This figure shows that the slope of pressure drop increases upon decreasing the sphere diameter. This result is expected because of using small spheres porous media generates high pressure drop due to more turbulence that results in high heat transfer coefficient as compared to large spheres porous media.

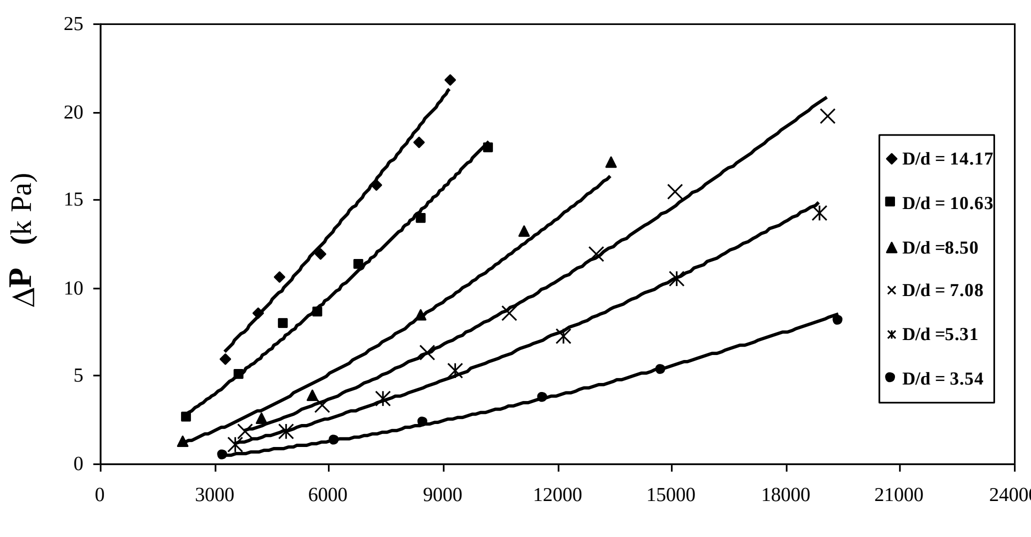

Fig. (7). Variation of pressure drop with Reynolds numbers for P.V.C. spheres of different diameters at the same conditions.

4.5. Correlations of the pressure drop results

By the same way the correlation between the dimensionless pressure drop and Reynolds number at various sphere diameters can be expressed in the following general form:

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = a_2 \operatorname{Re}_D{}^{b_2} \tag{18}$$

The values of the coefficients (a_2) and (b_2) are functions of the diameter ratio. The following relationships are obtained:

$$a_2 = c_3 e^{f_3 (D/d)}$$

$b_2 = f_4 (d / D) + f_5$

The final relationship becomes:

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = (1\times 10^{-4} e^{0.62(D/d)}) \operatorname{Re}_D{}^{(-0.045(D/d)+1.8)} \tag{19}$$

where: $2167 \le \operatorname{Re}_D \le 19400$ and $3.54 \le D/d \le 14.16$.

This relation gives errors in the range of ± 9.5% error.

4.6. Comparison between the present and previous work

When using packed tube the equation 16 is obtained. Elkady [4] reported the correlation between Nu_D and Re_D as follows:

$$Nu_D = 0.023(k_S / k_f)^{0.48} (D/d)^{0.66} \operatorname{Re}_D{}^{0.8} \operatorname{Pr}^{0.4} \tag{20}$$

where: $4141 \le \operatorname{Re}_D \le 14550$, $1.28 \le D/d \le 1.85$ and $6.54 \le k_S / k_f \le 2160$.

The present results shown in Eq. (16) are compared with Eq. (20) as shown in Fig. 8. The deviation between the present work and Elkady [4] at Re_D = 6000 is equal to - 8% but at Re_D = 10000 the deviation is equal to +14% due to different design parameters and experimental states.

The following table shows the difference between the present work and Elkady [4].

Table (3). The deviation between the present and Elkady [4] parameters.

Parameter	Present work	Elkady [4]
Test section	Steel tube	Copper tube
Test tube size	Di= 0.0425m, Do= 0.048 m and L=0.5 m	Di= 0.0204m, Do= 0.0225 m and L=0.7 m
D/d	3.54 - 14.16	1.28 - 1.85
kS/kf	0.24	6.54 – 2160
Porous media	P.V.C	Glass - Steel - P.V.C
Classification of work	Experimental	Experimental
Working Fluid	Water	Air
Porosity	0.38-0.42	0.53-0.6
ReD	2167-19400	4000-14000

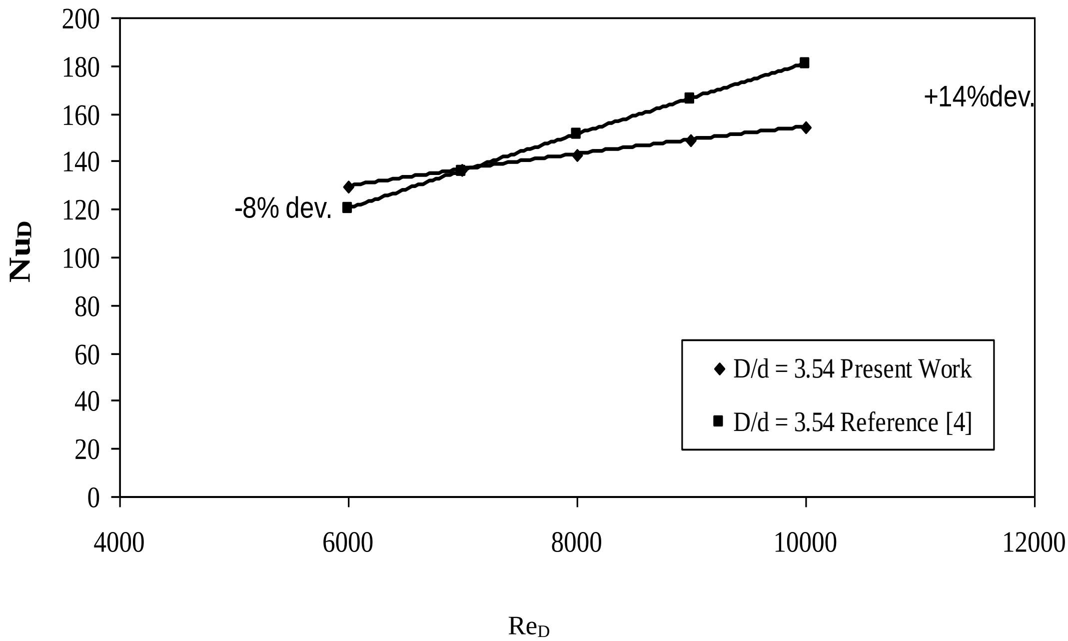

Fig. (8). Comparison between experimental work and Ref. [4] for packed bed tube.

5. Conclusions

The main conclusions from the present work are:

1. The heat transfer coefficient is improved by increasing Reynolds number (due to increasing the flow rate), resulting in an increase in Nusselt number.
2. The general correlation between Nusselt number and Reynolds number using porous material is presented by Eq. (16) for various diameter ratios and it has ± 7% error.
3. The general correlation between Nusselt number and Reynolds number without porous material is shown in Eq. (17).
4. Utilizing small sphere diameters are much better than using large diameters because they offer more turbulence and eddies which result in improving the heat transfer coefficient. On the other hand, higher pressure drop develops.
5. The results reveal that the heat transfer coefficient without using porous media is lower than the heat transfer coefficient when using porous media due to less turbulence, less eddies and approximately zero pressure drop on the test section.
6. The Nusselt number improves a factor of 3.8 to 6.3 times when using porous media for porous diameters from 0.012 to 0.003 m respectively as compared to the empty tube.
7. The general relationship between dimensionless pressure drop and Reynolds number at various sphere diameters is shown in Eq. (19) and it has a maximum error of ± 9.5%.

6. References

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تقدير معملي لعلاقات انتقال الحرارة بالأوساط المسامية

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(قُدم للنشر في ٢٠٠٧/٥/٢٧م؛ وقُبل للنشر في ٢٠٠٧/١١/٢٧م)

ملخص البحث. يهدف البحث إلى إيجاد العلاقة بين معامل انتقال الحرارة وسرعة سريان المائع في الأوساط المسامية. والوسط المسامي المستخدم هو وسط يتكون من كريات من مادة البولي فينيل كلوريد بأقطار مختلفة (من ٠,٠٠٣ إلى ٠,٠١٢ م). وقد تم وضع العلاقات في صورة لا بعدية على صورة علاقة بين رقم نوسلت ورقم رينولدز. والنسبة بين قطر أنبوب السريان إلى قطر كريات المادة المسامية. كذلك تم اختبار أنبوب فارغ بهدف مقارنة معامل انتقال الحرارة والفقد في الضغط خلال الأوساط المسامية والأوساط غير المسامية.

وقد خلصت الدراسة إلى النتائج التالية:

١-العلاقة بين رقم نوسلت ورقم رينولدز والنسبة بين قطر أنبوب السريان الى قطر الكريات كانت كالتالي:

$$Nu_D = 17.30\left(\frac{D}{d}\right)^{-0.77} \mathrm{Re}_D^{\,0.235\left(\frac{D}{d}\right)^{0.3}}$$

٢-العلاقة بين الفقد في الضغط ورقم رينولدز والنسبة بين قطر أنبوب السريان إلى قطر الكريات كالتالي:

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = (1\times10^{-4} e^{0.62(D/d)})\,\mathrm{Re}_D^{(-0.045(D/d)+1.8)}$$

٣-أدى استخدام أوساط مسامية إلى زيادة معامل انتقال الحرارة بنسبة تتراوح من ٣,٨ إلى ٦,٣ اعتمادًا على النسبة بين قطر أنبوب السريان وقطر الكريات عند تساوي رقم رينولدز .