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The Modern Syllogistic Method as a Tool for Engineering Problem Solving

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Abstract. We describe the steps and the main features of the modern syllogistic method, which is a very powerful technique of deductive inference. This method ferrets out from a set of premises all that can be concluded from it, with the resulting conclusions cast in the simplest or most compact form. We demonstrate the applicability of the method in a variety of engineering problems via five examples that illustrate its mathematical details and exhibit the nature of conclusions it can come up with. The method is shown to be particularly useful for detecting inconsistency within a set of given premises or hypotheses and it helps the engineer confront fallacy-based argumentation. The method is also demonstrated to yield fruitful results when combined with the safety technique known as fault-tree analysis. It is also used in selective deduction and in informed decision making.

Keywords: Deductive inference, Modern syllogistic method, Detecting inconsistencies, Real and perceived problems, Selective deduction, Informed decision making.

1. Introduction

One of the important traits of a successful engineer is logical thinking [1]. This trait can usually be acquired and mastered through appropriate training in deductive and inductive logic [2, 3]. Such training does not necessarily guarantee that a person can reason well or correctly, but a person knowledgeable about logic techniques is more likely to reason correctly than one who is unaware of them. In the past, smart individuals realized that they could think logically without the resort to the deductive logic techniques that were available to them, while dummy individuals failed to derive any benefit from these techniques, which were complex and cumbersome indeed. Traditional logicians applied deductive logic to contextual reasoning, and were deeply concerned with verbal fallacies. In its modern formal outlook, logic is a science of correct forms in which the study of such fallacies is irrelevant, and it has two distinctive branches of deduction and induction that are both essential as they play complementary rather than competitive roles in inference [3].

In this paper, we describe the steps, features and some applications to engineering problem solving of a very powerful technique for deductive inference, which we call "the modern syllogistic method". The first popular description of this method is given by Brown [4]. Later presentations of the method are given by Gregg [5] and Rushdi and Al-Shehri [6]. The great advantage of the method is that it ferrets out from a given set of premises all that can be concluded from this set, and it casts these conclusions in the simplest or most compact form.

The remainder of this paper is organized as follows. Section 2 outlines the steps of the modern syllogistic method, while Section 3 lists its main features. Section 4 illustrates some applications of the method to engineering problem solving in terms of five examples. Example 1 presents typical deductions by the method in the context of a problem of mechanism testing or troubleshooting. Examples 2 and 3 demonstrate how the method can test hypotheses or detect inconsistencies within a set of premises. This feature is very useful for the engineer in his role as a problem solver because he can avoid falling into the trap of solving a perceived problem, which is a problem thought to be correctly defined while, in fact, it is not. The same feature is also necessary for the engineer in his role as an argumenter, because it assists him to avoid being deceived by those who use inconsistent premises to validly deduce false conclusions, no matter how irrelevant they are. Example 4 combines the method with the well known safety technique of fault-tree analysis, thereby producing substantially fruitful results. The ramifications of such a combination are far reaching and warrant further exploration. Example 5 presents a case of selective deduction and informed decision making. Section 5 concludes the paper.

2. Steps of the Modern Syllogistic Method

The modern syllogistic method has the following steps:

1. Each of the premises is converted into the form of a formula equated to 0 (which we call an equational form), and then the resulting equational forms are combined together into a single equation of the form $f = 0$. If we have n logical equivalence relations of the form:

$$T_i \equiv Q_i \,, \qquad 1 \le i \le n \,, \tag{1}$$

Then they are set in the equational form:

$$T_i \,\overline{Q}_i \vee \overline{T}_i \, Q_i = 0 \,, \qquad 1 \le i \le n. \tag{2}$$

We may also have ($m - n$) logical implication (logical inclusion) relations of the form:

$$T_i \rightarrow Q_i \,, \qquad (n+1) \le \mathrm{i} \le m \,. \tag{3}$$

These relations symbolize the statements "$If\ T_i\ then\ Q_i$" or equivalently "$T_i\ if\ only\ Q_i$". The conditions in (3) can be set into the equational form:

$$T_i \,\overline{Q}_i = 0, \qquad (n+1) \le \mathrm{i} \le m. \tag{4}$$

The totality of m premises in Eqs. (1) and (3) finally reduce to the single equation $f = 0$, where f is given by [7]:

$$f = \bigvee_{i=1}^{n} (T_i \,\overline{Q}_{\mathrm{i}} \vee \overline{T}_i \, Q_i) \vee \bigvee_{i=(n+1)}^{m} T_i \,\overline{Q}_i. \tag{5}$$

Equations (1) and (3) represent the dominant forms that premises can take. Other less important forms are discussed by Klir and Marin [8] and can be added to Eq. (5) when necessary.

2. The function f in Eq. (5) is rewritten as a complete sum (Blake canonical form), i.e. as a disjunction of all the prime implicants of f. There are many manual and computer algorithms for developing the complete sum of a switching function f [4, 9-11]. Most of these algorithms depend on two logical operations: (a) Consensus generation (or equivalently multiplying a product of sums into a sum of products), and (b) absorption.
3. Suppose the complete sum of f takes the form:

$$f = \bigvee_{i=1}^{\ell} P_i = 0\,, \tag{6}$$

where P_i is the i th prime implicant of f. Equation (6) is equivalent to the set of equations:

$$P_i = 0\,, \qquad 1 \le i \le \ell. \tag{7}$$

Equation (7) states in the simplest equational form all that can be concluded from the original premises. The conclusions in Eq. (7) can also be cast into the implication form. Suppose P_i is given as a conjunction of uncomplemented literals X_{ij} and complemented literals; $\overline{Y}_{ij}$, i.e.

$$P_i = \bigwedge_{j=1}^{r} X_{ij} \wedge \bigwedge_{j=1}^{s} \overline{Y}_{ij}\,, \qquad 1 \le i \le \ell\,, \tag{8}$$

then, Eq. (7) can be rewritten as:

$$\bigwedge_{j=1}^{r} X_{ij} \rightarrow \overline{(\bigwedge_{j=1}^{s} \overline{Y}_{ij})}, \quad 1 \le i \le \ell\,, \tag{9}$$

or as:

$$\bigwedge_{j=1}^{r} X_{ij} \rightarrow \bigvee_{j=1}^{s} Y_{ij}\,, \quad 1 \le i \le \ell. \tag{10}$$

3. Important Features of the Modern Syllogistic Method

1. The modern syllogistic method produces all possible consequents (since CS(f) is a disjunction of all the prime implicants of f), and it casts these consequents in the most compact form (since all the implicants in CS(f) are prime ones). If any implicant (whether it is prime or not) of f is equated to 0, then the result is a true consequent (albeit not necessarily in the most compact form) [4].
2. To test the truth of any claimed consequent based on a given set of premises, one just needs to cast these claimed consequents in the form of a disjunction of terms equated to 0, and check to see if each of these terms subsumes (at least) one of the prime implicants in CS(f) derived for the set of premises.
3. The modern syllogistic method encompasses a complete set of inference rules, and constitutes a complete system of truth-functional logic, in the sense that it permits the construction of a formal proof of validity for any valid truth-functional argument [6].
4. The modern syllogistic method has a built-in capability of detecting the existence of inconsistency within a given set of premises, The method will alert its user to the existence of concealed inconsistencies by producing CS(f)=1. Once this happens, the user should refrain from making any conclusion, and should revise his set of premises to change it into a consistent one.
5. The modern syllogistic method can be used in detecting and invalidating certain purported arguments or formal fallacies, such as the converse fallacy (the fallacy of affirming the consequent) or the inverse fallacy (the fallacy of denying the antecedent).
6. The modern syllogistic method is very useful in the case of selective deduction [12], which is deduction with the knowledge of certain information or restrictions, or the lack thereof, about some of the pertinent variables. The method handles selective deduction by:

a) either selecting the appropriate subset of the set of prime implicants in Eq. (6) or by obtaining the appropriate conjunctive eliminant [4] or meet derivative [13] of f in Eq. (5), and then casting it in complete-sum form [12], or
b) restricting the values of appropriate variables by assigning each of them one of the constant values 0 or 1.

7. As a formal technique of logic, the modern syllogistic method concerns itself only with the form of its premises and consequents and has nothing to do with their subject matter. It is up to the user of the method to use plausible heuristics to formulate the premises and interpret the consequents. The intervening task of going from the formal premises to the formal consequents is tackled in a completely algorithmic fashion by the method. By contrast, the heuristics required of the user are fallible, involve some linguistic and verbal elements, and cannot be replaced by exact recipes or algorithms.

4. Examples

4.1. Example 1

This example is posed as a problem by Brown [4]. It illustrates the mathematical details of the method and the clear insight it provides in going from intricate premises to much simplified consequents. Consider the following situation. The state of a mechanism under test is shown by five indicators, labeled A, B, C, D and E. After watching the indicators for a long time, an observer characterizes the mechanism as follows:

a) If A or D is on (but not both), then C is on.
b) Looking just at C, D and E, the number of on-indicators is always odd.
c) If E is off, then A and D are both off.
d) If B and C are both on, then E is on.
e) At least one of the following conditions always exists:
   i) A on.
   ii) C off.
   iii) D on.

Express the prime consequents in clausal form.

Conditional form	Equational form
$A \oplus D \rightarrow C$	$A\overline{D}\,\overline{C} \vee \overline{A}D\overline{C} = 0$
$C \oplus D \oplus E$	$\overline{C}\,\overline{D}\,\overline{E} \vee C D \overline{E} \vee C \overline{D} E \vee \overline{C} D E = 0$
$\overline{E} \rightarrow \overline{A}\,\overline{D}$	$\overline{E} A \vee \overline{E} D = 0$
$B C \rightarrow E$	$B C \overline{E} = 0$
$A \vee \overline{C} \vee D$	$\overline{A} C \overline{D} = 0$

The given data are, therefore, equivalent to the propositional equation $f = 0$, where f is given by:

$$f = A\overline{D}\,\overline{C} \vee \overline{A}D\overline{C} \vee \overline{C}\,\overline{D}\,\overline{E} \vee C D \overline{E} \vee C \overline{D} E \vee \overline{C} D E \vee \overline{E} A \vee \overline{E} D \vee B C \overline{E} \vee \overline{A} C \overline{D} \tag{11}$$

The complete sum for f (the Blake canonical form for f) is obtained by the improved Tison method [11] as shown in Fig. 1 in which consensi are formed with respect to each of the four variables A, C, D and E respectively. No consensi are formed with respect to the variable B since it is a monoform variable. Each step of consensus generation is followed by a step of absorption in which a term is absorbed by another if the former subsumes the latter (i.e., if the set of literals for the absorbed term is a superset of the literals for the absorbing term). In Fig. 1, encircled terms are those absorbed, while those surviving absorption are set in bold. The formula expressing f gradually evolves as:

A

	$\bar{A}\bar{C}D$	$\bar{A}C\bar{D}$	$\bar{C}\bar{D}\bar{E}$ $CD\bar{E}$ $C\bar{D}E$ $\bar{C}DE$ $D\bar{E}$ $BC\bar{E}$
$A\bar{E}$ $A\bar{C}\bar{D}$	$\bar{C}D\bar{E}$ –	$C\bar{D}\bar{E}$ –	

C

	$\bar{C}\bar{D}\bar{E}$	$\bar{C}DE$	$A\bar{C}\bar{D}$	$\bar{A}\bar{C}D$	
$C\bar{D}\bar{E}$	$\bar{D}\bar{E}$	–	$A\bar{D}\bar{E}$	–	$D\bar{E}$ $A\bar{E}$
$BC\bar{E}$	$B\bar{D}\bar{E}$	–	$AB\bar{D}\bar{E}$	$\bar{A}BD\bar{E}$	
$C\bar{D}E$	–	–	$A\bar{D}E$	–	
$\bar{A}C\bar{D}$	$\bar{A}\bar{D}\bar{E}$	–	–	–	

D

	$\bar{D}\bar{E}$	$A\bar{C}\bar{D}$	$A\bar{D}E$	$C\bar{D}E$	$\bar{A}C\bar{D}$	
$\bar{C}DE$	–	$A\bar{C}E$	$A\bar{C}E$	–	–	$A\bar{E}$ $BC\bar{E}$
$\bar{A}\bar{C}D$	$\bar{A}\bar{C}\bar{E}$	–	–	–	–	
$D\bar{E}$	$\bar{E}$	$A\bar{C}\bar{E}$	–	–	$\bar{A}C\bar{E}$	

E

	$\bar{E}$	
$A\bar{C}E$ $C\bar{D}E$ $A\bar{D}E$ $\bar{C}DE$	$A\bar{C}$ $C\bar{D}$ $A\bar{D}$ $\bar{C}D$	$\bar{A}\bar{C}D$ $A\bar{C}\bar{D}$ $\bar{A}C\bar{D}$

Fig. (1). Derivation of the complete sum for in Eq. (12) by the imporoved Tison method.

$$f = A\,\overline{E} \vee A\,\overline{C}\,\overline{D} \vee \overline{A}\,\overline{C}\,D \vee \overline{A}\,C\,\overline{D} \vee \overline{C}\,\overline{D}\,\overline{E} \vee C\,D\,\overline{E}$$

$$\vee C\,\overline{D}\,E \vee \overline{C}\,D\,E \vee D\,\overline{E} \vee B\,C\,\overline{E}$$

$$= C\,\overline{D}\,\overline{E} \vee B\,C\,\overline{E} \vee C\,\overline{D}\,E \vee \overline{A}\,C\,\overline{D} \vee \overline{C}\,\overline{D}\,\overline{E} \vee \overline{C}\,D\,E$$

$$\vee A\,\overline{C}\,\overline{D} \vee \overline{A}\,\overline{C}\,D \vee D\,\overline{E} \vee A\,\overline{E}$$

$$= \overline{C}\,D\,E \vee \overline{A}\,\overline{C}\,D \vee D\,\overline{E} \vee \overline{D}\,\overline{E} \vee A\,\overline{C}\,\overline{D} \vee A\,\overline{D}\,E$$

$$\vee C\,\overline{D}\,E \vee \overline{A}\,C\,\overline{D} \vee A\,\overline{E} \vee B\,C\,\overline{E}$$

$$= A\,\overline{C}\,E \vee C\,\overline{D}\,E \vee A\,\overline{D}\,E \vee \overline{C}\,D\,E \vee \overline{E}$$

$$\vee \overline{A}\,\overline{C}\,D \vee A\,\overline{C}\,\overline{D} \vee \overline{A}\,C\,\overline{D}$$

$$= \overline{E} \vee A\,\overline{C} \vee A\,\overline{D} \vee C\,\overline{D} \vee \overline{C}\,D \tag{12}$$

where the last formula stands for $CS(f)$, i.e. it is a disjunction of all the prime implicants of f. Equation (12) is equivalent to:

$$\overline{E} = 0, \tag{13a}$$

$$A\overline{C} \vee A\overline{D} = 0, \qquad \{A \rightarrow C\,D\} \tag{13b}$$

$$C\,\overline{D} \vee \overline{C}\,D = 0, \qquad \{C \equiv D\} \tag{13c}$$

We therefore conclude that indicator E is always on, and if A is on, then both C and D are on, while indicators D and C always assume the same instantaneous value, i.e. they are both on or they are both off. For indicator B, we lack any kind of information, though our premises suggest that we have something to tell about B. To verify the solution, we can ascertain that any term in the equational forms of the premises subsumes at least one prime implicant in $CS(f)$, and that if the consequents in Eq. (13) are imposed on the premises, each of the premises turns into a tautology.

4.2. Example 2

The scenario discussed in this example is a case study about differentiating a perceived problem from a real one [14]. There is a toxic discharge from a chemical plant into a nearby river. Due to a summer drought, the discharge might no longer be sufficiently dilute to be safe to aquatic life. In fact, the discharge is believed to be responsible for an unusually high number of dead fish that is turning up in the river. An engineer is called upon to design a million-dollar waste treatment facility to reduce the toxic chemical concentration by a factor of 10. However, his investigations indicate that dead fish are appearing at the same unusually high rate everywhere, not just downstream of the plant. Let us introduce the propositional variables:

T = The plant discharges toxic chemicals into the river.
D = The toxic chemicals flow downstream.
U = The toxic chemicals flow upstream.
N = Fish die downstream.
P = Fish die upstream.

We now test the hypothesis that fish die if and only if there are toxic chemicals. Our premises are:

Clausal form	Conditional form
T	$\overline{T}=0$
$T \rightarrow D$	$T\,\overline{D}=0$
$T \rightarrow \overline{U}$	$T\,U=0$
$N \equiv D$	$N\,\overline{D} \vee \overline{N}\,D=0$
$P \equiv U$	$P\,\overline{U} \vee \overline{P}\,U=0$
N	$\overline{N}=0$
P	$\overline{P}=0$

These premises combine to give the function:

$$f=\overline{T} \vee T\,\overline{D} \vee T\,U \vee N\,\overline{D} \vee \overline{N}\,D \vee P\,\overline{U} \vee \overline{P}\,U \vee \overline{N} \vee \overline{P}, \tag{14}$$

whose complete sum is:

$$CS(f)=1, \tag{15}$$

which leads to the contradiction 1=0. This means that the set of premises is inconsistent. There is no way to make all the premises true at the same time. Moreover, the given set of premises validly yields any conclusion, no matter how irrelevant [3]. In the above situation, the remedy for the inconsistency is to discard (at least) one of the given premises. The engineer must abandon the premises $N \equiv D$ and $P \equiv U$ which arise from the notion that his factory's chemicals are the real fish killer. Further investigations can lead to the real culprit which turns out to be a certain type of fungus in the given scenario [14].

We have deliberately chosen the current example to be a small one, so that the reader might easily convince himself about the existence of inconsistency among the premises by just viewing their verbal statements and without the resort to the logic technique. In more sophisticated and involved scenarios, inconsistency within a set of premises is much harder to detect and is intricately concealed and hidden. The engineer cannot usually handle such scenarios bare-handed, but he will hopefully be able to tackle them when armed with the present powerful method.

4.3. Example 3

This example does not deal with an engineering problem per se, though it handles a problem of concern to many engineers. It demonstrates how an engineer can confront illogical thinking and fallacious argumentation. Consider the situation of a retiring engineer who has served his company for two consecutive periods of time. In the first period, the terms of employment were decided by an old set of statutes (O), but in the second period the company switched to a new set of statutes (N). Each set of statutes is self consistent and strives to achieve its own sense of justice. According to the old statutes, the end-of-service gratuity is a full-month salary (F) per year of service, but this gratuity is only a half-month salary (H) per year of service in the new statutes. Also, the new statutes set an upper limit (L) on the gratuity, while in the old statutes there is no such limit. If the engineer's service is considered continuous (C), the engineer receives a total gratuity (T) for his total service according to his initial contract based on the old statutes. Otherwise, he receives two split gratuities (S), one covering his first period of service and based on the old statutes, and another fresh one covering his second period and based on the new statutes. We now formalize the aforementioned premises as follows:

Clausal form	Conditional form
$O \to F\,\overline{L}$	$O\,\overline{F} \vee O\,L = 0$
$N \to H\,L$	$N\,\overline{H} \vee N\,\overline{L} = 0$
$C \to O\,T$	$C\,\overline{O} \vee C\,\overline{T} = 0$
$\overline{C} \to N\,S$	$\overline{C}\,\overline{N} \vee \overline{C}\,\overline{S} = 0$
$N \equiv \overline{O}$	$NO \vee \overline{N}\,\overline{O} = 0$
$S \equiv \overline{T}$	$S\,T \vee \overline{S}\,\overline{T} = 0$

These premises are now combined into a single equation of the form:

$$f = O\,\overline{F} \vee O\,L \vee N\,\overline{H} \vee N\,\overline{L} \vee C\,\overline{O} \vee C\,\overline{T} \vee \overline{C}\,\overline{N} \vee \overline{C}\,\overline{S} \vee N\,O \vee \overline{N}\,\overline{O} \vee S\,T \vee \overline{S}\,\overline{T} = 0, \tag{16}$$

The complete sum of f is obtained via the improved Tison method [11]:

$$\begin{aligned} CS(f) = {} & O\,\overline{F} \vee O\,L \vee N\,\overline{H} \vee N\,\overline{L} \vee C\,\overline{O} \vee C\,\overline{T} \vee \overline{C}\,\overline{N} \vee \overline{C}\,\overline{S} \vee NO \\ & \vee \overline{N}\,\overline{O} \vee S\,T \vee \overline{S}\,\overline{T} \vee \overline{C}\,T \vee C\,S \vee \overline{O}\,\overline{S} \vee \overline{O}\,T \vee \overline{N}\,\overline{T} \\ & \vee \overline{N}\,S \vee C\,\overline{F} \vee C\,L \vee C\,N \vee \overline{N}\,\overline{F} \vee \overline{N}\,L \vee \overline{F}\,\overline{S} \vee \overline{F}\,T \\ & \vee L\,\overline{S} \vee L\,T \vee N\,\overline{S} \vee N\,T \vee \overline{C}\,\overline{H} \vee \overline{O}\,\overline{H} \vee \overline{T}\,\overline{H} \vee \overline{F}\,\overline{H} \\ & \vee L\,\overline{H} \vee \overline{C}\,\overline{L} \vee \overline{O}\,\overline{L} \vee \overline{T}\,\overline{L} \vee \overline{F}\,\overline{L} \vee \overline{C}\,O \vee \overline{T}\,O = 0. \end{aligned} \tag{17}$$

Equation (17) ferrets out all the prime consequents that can be deduced from the original premises, and they are of a relatively huge number indeed. Each of these consequents makes sense in the view of the original premises. For example, $L\,\overline{H} = 0$ indicates that $L \to H$, i.e. if the gratuity has a limit, it is at the half-month rate. Note that if one adds a premise about the continuity of service, by asserting either C or $\overline{C}$, then more tangible and decisive conclusions can be reached. However, the "clever" lawyers at the so called "legal" department of the company deliberately refuse to settle the question of continuity and arbitrarily decide to assert T and N as additional premises. These new premises appear in equational form as $\overline{T} = 0$ and $\overline{N} = 0$, and when the disjunction comprising the function f is augmented by them, the formula of f includes now the three terms $N\,T$, $\overline{T}$, and $\overline{N}$ which sum up to 1, and hence:

$$CS(f) = 1,$$

which leads to the contradiction 1 = 0. This means that the total set of premises is inconsistent, and hence it is totally worthless as a basis of deduction [3]. Such a set of inconsistent premises can be used to validly yield any conclusion, no matter how irrelevant. In fact, inconsistent premises can be used to conclude simultaneously any proposition D ($\overline{D} = 0$) and its denial $\overline{D}$ ($D = 0$), since both the terms $\overline{D}$ and D subsume (are included in) the term 1. The "clever" lawyers are now at leisure to forward any unfair decision and disguise it as a valid consequent of their "legal" premises. The engineer should, if he can, insist on (a) showing that there is inconsistency within the given premises, (b) refusing to deduce anything from these premises, and (c) requesting a revision of the premises to ensure their consistency and truth.

4.4. Example 4

Fault tree analysis [15, 16] is a deductive safety analysis technique, which starts at a hazard event and traces backwards to find the events which caused it. The analysis is represented in a diagrammatic form, with symbols representing the events, and logic gates showing the relationships between the events. A fault tree is built from a top level undesirable event simply called the top event. The top level event is decomposed using a series of guidelines which help to identify the contributing factors leading to the undesirable event. For each event in the tree, the immediate causes for this event are identified and determined to be either necessary or sufficient causes. If all identified cases are preconditions for the occurrence of the higher level event, these are considered to be necessary causes and they are conjoined in the fault tree diagram using a logical AND-gate. If individual causes can each result in the higher level event, these are deemed to be sufficient causes, and the relationship between these causes is represented in the fault tree diagram with an OR-gate. There are other gates which are used to represent other causal relationships, such as EXCLUSIVE-OR and PRIORITY-AND.

Many practical fault trees are very complex and involve literally thousands of gates and events. However, we will deal here with a very small example of a portable kerosene heater. This heater has the potential problems of being mistakenly filled with an improper fuel, tipping over, or causing carbon monoxide buildup [14]. These potential problems correspond respectively to the top events or consequences of explosion (X), fire (F), and asphyxiation (A). Figure 2 depicts a combination of three fault trees (called a fault forest) for these three top events. Mathematically, the top events are given in terms of the basic events defined in Fig. 2 as:

$$A \equiv Y\,\overline{S}\,(E\,C \vee \overline{E}), \tag{18a}$$

$$F \equiv Y\;(E\,C \vee \overline{E})\;T, \tag{18b}$$

$$X \equiv Y\,E\,\overline{C}, \tag{18c}$$

where

$$Y \equiv U\,O. \tag{18d}$$

Equations (18a-18c) constitute our set of premises, and can be combined into a single equation of the form:

$$g = (A \oplus Y\,\overline{S}\,(E\,C \vee \overline{E})) \vee (F \oplus Y\,(E\,C \vee \overline{E})\,\mathrm{T}) \vee (X \oplus Y\,E\,\overline{C}) = 0. \tag{19}$$

The function g in Eq. (19) can now be recast into complete-sum form (e.g. by the improved Tison method), so that Eq. (19) becomes:

$$CS(g) = (\overline{A}\,Y\,\overline{S}\,C \vee \overline{A}\,Y\,\overline{S}\,\overline{E} \vee \overline{F}\,Y\,T\,C \vee \overline{F}\,Y\,T\,\overline{E} \vee \overline{X}\,Y\,\overline{C}\,E) \vee (A\overline{Y} \vee A\,S \vee A\,\overline{C}\,E \vee F\,\overline{Y} \vee F\,\overline{T} \vee F\,\overline{C}\,E \vee X\,\overline{Y} \vee X\,\overline{E} \vee X\,C) \vee (F\,X \vee A\,X \vee F\,\overline{S}\,\overline{A} \vee A\,\overline{F}\,T \vee \overline{F}\,T\,Y\,\overline{X} \vee \overline{A}\,Y\,\overline{S}\,\overline{X}) = 0. \tag{20}$$

The prime consequents in Eq. (20) are partitioned with parentheses into three sets:

a) The first set is a set of prime consequents representing:

$$\overline{A}\,CS(A) \vee \overline{F}\,CS(F) \vee \overline{X}\,CS(X) = 0, \tag{21}$$

where $CS(A)$, for example, represents $CS(Y\,\overline{S}\,(E\,C \vee \overline{E})) = Y\,\overline{S}\,(C \vee \overline{E})$. The terms in this set correspond to the minimal cutsets of the individual fault trees. For example, the prime consequent $\overline{A}\,Y\,\overline{S}\,C = 0\,(Y\,\overline{S}\,C \rightarrow A)$ indicates that the basic events Y, $\overline{S}$ and C constitute a minimal cutset for the top event A, i.e. the simultaneous occurrence of theses three events causes A to occur, but if one of them is missing, A does not occur. Note that some, but not all, of the terms belonging to this first set are visually obvious from the fault trees themselves. To obtain all of these terms, some cutset enumeration technique is needed [17, 18].

b) The second set is a set of prime consequents representing:

$$A\ CS(\overline{A}) \vee F\ CS(\overline{F}) \vee X\ CS(\overline{X}) = 0, \tag{22}$$

where $CS(\overline{A})$, for example, represents $CS(\overline{Y\,\overline{S}\,(E\,C \vee \overline{E})}) = \overline{Y} \vee S \vee \overline{C}\,E$. The terms in this set correspond to what might be called "minimal tie sets" of the individual fault trees. For example, the prime consequent $A\overline{Y} = 0\ (\overline{Y} \rightarrow \overline{A})$ indicates that non-occurrence of the basic event Y guarantees the non-occurrence of the top event A. The terms in this second set are not directly obtainable from the fault trees, as they require an algebraic process of negation or complementation associated with complete-sum generation.

c) The third set is a set representing:

$$F\,X \vee A\,X \vee F\,\overline{S}\,\overline{A} \vee A\,\overline{F}\,T \vee \overline{F}\,T\,Y\,\overline{X} \vee \overline{A}\,Y\,\overline{S}\,\overline{X} = 0. \tag{23}$$

This third set has prime consequents involving literals for more than one top event. These consequents are quite hidden within the initial premises. They include two consequents of particular interest, namely, $F\,X = 0$ and $A\,X = 0$. These two consequents tell us what information we have about top events in the absence of information about basic events: (a) Fire and explosion cannot occur simultaneously, and (b) Asphyxiation and explosion cannot occur simultaneously.

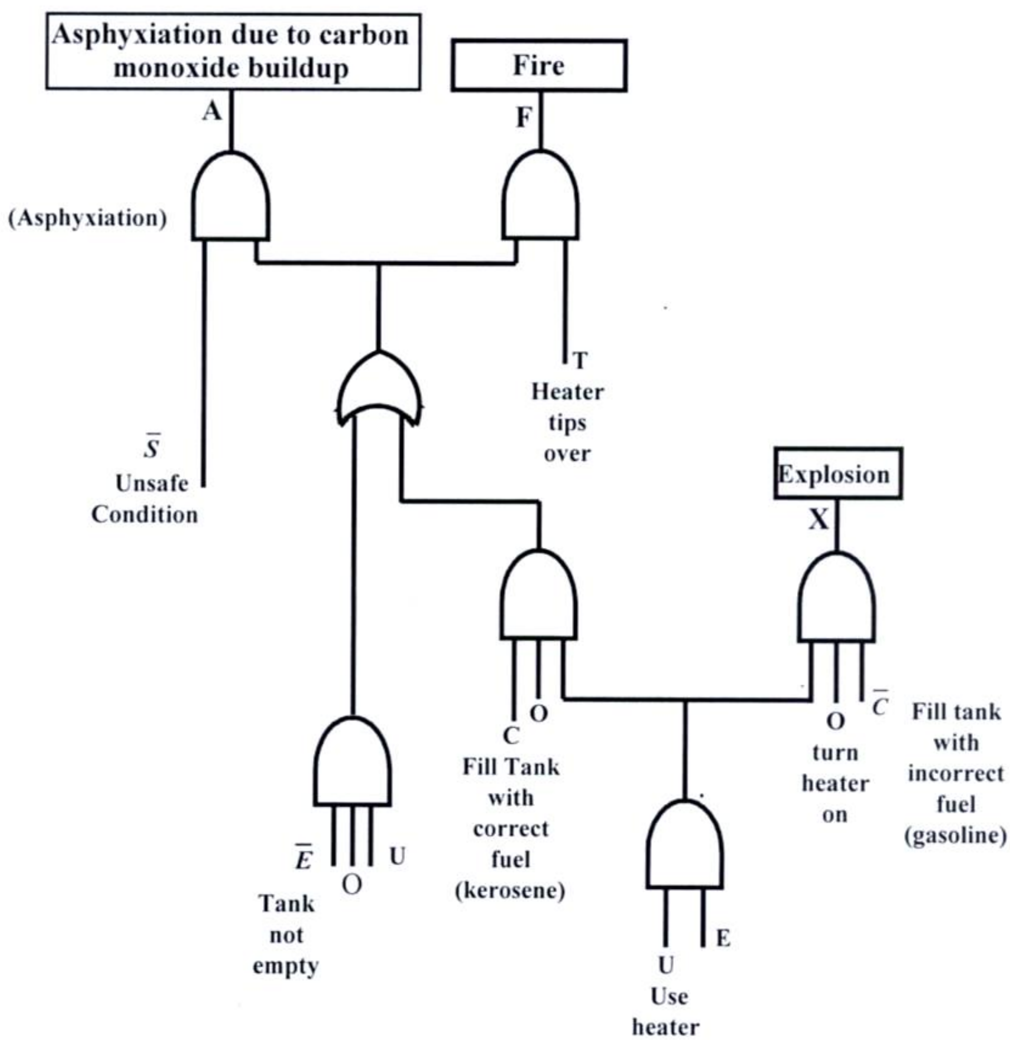

Fig. (2). A fault forest associated with a kerosene heater.

4.5. Example 5

Your company is one of two leading companies that are having almost equal market shares for a certain popular product. To obtain a competitive edge over your rival, you want to join arms with three smaller companies X, Y and Z, so as to form a new consortium of companies or a mega-company. However, your main competitor has exactly the same idea, and plans to establish a similar alliance with three companies A, B and C which are essentially of similar size, technical expertise, and resources as those of companies X, Y and Z. Due to certain market forces government regulations, and conflicts of interests, the following restrictions exist about the participation of the six small companies in the two alliances:

1. If neither A, B nor C joins the rival alliance, then Y and Z join yours, but X does not.
2. If A joins the rival alliance together with either B or C or both, then Y does not join your alliance, and either X does not join it or Z joins it.
3. If B joins your competitors but A does not, or C joins them but B does not, then both X and Y join you, or neither X nor Z does.
4. If C allies with your rival together with A or B or both, or if neither A nor C joins, then either X does not join you, or Y does but Z does not.
5. If A joins the rival alliance but B does not, then X does not join you or Z does.

Now, we pose the following three questions:

a) In the absence of any information about the participation of companies A, B and C in the rival alliance, what can you conclude about the participation of companies X, Y, and Z in your alliance?

b) You are currently contemplating awarding a contract to one of the X, Y, and Z companies, and you believe that the company awarded that contract is definitely guaranteed to join your alliance. Which company would you choose to maximize the participation in your alliance?

c) If you implement the action suggested in (b), and given no further information, what can you conclude about participation of the A, B, and C companies in the rival alliance? Is your alliance bigger than the rival one?

To answer these questions, we formulate the given premises (1-5) as follows:

1. $\overline{A}\,\overline{B}\,\overline{C} \quad \rightarrow \quad \overline{X}\,Y\,Z,$
2. $A(B \vee C) \quad \rightarrow \quad \overline{Y}(\overline{X} \vee Z),$
3. $\overline{A}B \vee \overline{B}C \quad \rightarrow \quad X\,Y \vee \overline{X}\,\overline{Z},$
4. $C(A \vee B) \vee \overline{A}\,\overline{C} \quad \rightarrow \quad \overline{X} \vee Y\,\overline{Z},$
5. $A\,\overline{B} \quad \rightarrow \quad \overline{X} \vee Z.$

The premises combine into a single equation $f = 0$ where:

$$f = \overline{A}\,\overline{B}\,\overline{C}\,(X \vee \overline{Y} \vee \overline{Z}) \vee A\,(B \vee C)\,(Y \vee X\,\overline{Z}) \vee (\overline{A}\,B \vee \overline{B}\,C)\,(X\,\overline{Y} \vee \overline{X}\,Z)$$

$$\vee\, (A\,C \vee B\,C \vee \overline{A}\,\overline{C})\; X\; (\overline{Y} \vee Z) \vee A\,\overline{B}\,X\,\overline{Z}, \tag{24}$$

The function has the complete sum [4, 12]:

$$CS(f) = A\,C\,X \vee A\,X\,\overline{Z} \vee A\,C\,Y \vee A\,\overline{B}\,C\,Z \vee A\,B\,Y \vee \overline{A}\,X\,\overline{Y} \vee C\,X\,\overline{Y}$$

$$\vee\, X\,\overline{Y}\,\overline{Z} \vee \overline{A}\,\overline{Y}\,Z \vee \overline{B}\,C\,\overline{Y}\,Z \vee \overline{A}\,C\,\overline{X}\,Z \vee C\,\overline{X}\,Y\,Z \tag{25}$$

$$\vee\, \overline{B}\,C\,\overline{X}\,Z \vee \overline{A}\,\overline{B}\,\overline{C}\,X \vee \overline{B}\,\overline{C}\,X\,\overline{Z} \vee \overline{A}\,\overline{B}\,\overline{C}\,\overline{Y} \vee \overline{A}\,\overline{B}\,\overline{C}\,\overline{Z}$$

$$\vee\, \overline{A}\,B\,Z \vee B\,C\,X\,Z \vee B\,Y\,Z \vee \overline{A}\,\overline{C}\,X\,Z.$$

In (a), we lack any information about A, B and C. Therefore, we eliminate the variables A, B and C by deleting every prime implicant involving A, B or C from the equation *CS(f)* = 0. This produces the result:

$$X\,\overline{Y}\,\overline{Z}=0, \tag{26}$$

which can be stated in the conditional form:

$$X \rightarrow Y \vee Z, \tag{27a}$$

which means that (in the absence of information about the rival participation) if company X joins you, then company Y or company Z or both will also join you. Equation (26) can also be stated in either of the equivalent conditional forms:

$$\overline{Y} \rightarrow \overline{X} \vee Z, \tag{27b}$$

$$\overline{Z} \rightarrow \overline{X} \vee Y, \tag{27c}$$

which we will not explore further since they are not pertinent to the decision requested in question (b). In fact, to answer question (b), let us consider what happens if company Y is selected by assigning the value 1 to Y in Eq. (26). This reduces Eq. (26) to the identity 0=0, which says that if Y is selected, there will be no information of what will become of X and Z. Similarly, if company Z is selected, we obtain no information about the participation of X and Y. Therefore, it is prudent to award the contract to company X so as to guarantee its participation, since this will trigger the participation of at least one of the two other companies, and hence you get two or three companies joining your alliance. If you grant the contract to either Y or Z, you guarantee the participation of only the single company awarded the contract.

To answer question (c) assuming that your decision in (b) is to ensure the participation of X, we restrict X to the value 1 in Eq. (25), to obtain (after the absorption of subsuming terms):

$$CS(f)=A\,C \vee A\,\overline{Z} \vee A\,B\,Y \vee \overline{A}\,\overline{Y} \vee C\,\overline{Y} \vee \overline{Y}\,\overline{Z} \vee \overline{A}\,\overline{B}\,\overline{C}$$

$$\vee\ \overline{B}\,\overline{C}\,\overline{Z} \vee \overline{A}\,B\,Z \vee B\,C\,Z \vee B\,Y\,Z \vee \overline{A}\,\overline{C}\,Z. \tag{28}$$

Now we locate the prime implicants in Eq. (27) that involve variables A, B and C only. This gives us the results:

$$A\,C=0, \tag{29}$$

$$\overline{A}\,\overline{B}\,\overline{C} = 0, \tag{30}$$

which means that in the absence of any further information, the rival alliance will not be joined simultaneously by A and C, though it will be joined by at least one of the three companies A, B and C. This means that your competitor recruits one or two companies to join his alliance. Your alliance, however, is joined by two or three companies. Your alliance is most likely bigger than, or at least equal to, your competitor's alliance.

5. Conclusions

This paper describes the modern syllogistic method, which ferrets out from a given set of premises all the consequents that can be concluded from this set, and casts these consequents in the simplest or most compact form. The modern syllogistic method can deal with arguments of many varieties on many different topics, but it is restricted herein to the engineering subject matter. We believe that the modern syllogistic method can serve as a useful and powerful tool for the engineer, as it can help him reason well and correctly about his specific discipline. Due to space limitations, the paper presents only a quick glimpse of the many possible engineering applications of the method. Notable among the ones excluded here is the application of the method to problems of controllability and observability in automatic control [19] and to the resolution of engineering ethical dilemmas [20].

We are currently investigating the utility of the modern syllogistic method in avoiding the trap of illusory inference, which is a class of erroneous deductions that are compelling but invalid [21, 22]. We are also trying to make use of the modern syllogistic method in the study of enthymemes, which are arguments, or chains of argumentation, with one or more missing (implicit) premises or conclusions [23]. Our target is to devise a general technique to fill in missing premises in an enthymeme subject to some reasonable criterion of acceptability. Work on this problem is promising, since it is quite related to the well-developed problem of finding a best-fit extension of a partially defined Boolean function [24].

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الطريقة الاستدلالية الحديثة كأداة لحل المسائل الهندسية

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(قُدم للنشر في ٢٠/٦/٢٠٠٧م؛ وقُبل للنشر في ٦/١١/٢٠٠٧م)

ملخص البحث. نقدم وصفًا للخطوات والملامح الأساسية للطريقة الاستدلالية الحديثة التي تمثل أسلوبًا قويًّا للاستدلال الاستنباطي. تستخرج هذه الطريقة من مجموعة من المقدمات كل ما يمكن استنتاجه منها، وتصوغ الاستنتاجات الناجمة في أبسط صورة ملمومة. نبين إمكانية تطبيق هذه الطريقة في مسائل هندسية متنوعة باستخدام خمسة أمثلة توضح التفصيلات الرياضية للطريقة، كما تفصح عن طبيعة الاستنتاجات التي يمكن أن تأتي بها. يتم بيان أن هذه الطريقة تفيد بصفة خاصة في اكتشاف وجود عدم انسجام في مجموعة من المقدمات أو الافتراضات ومن ثم فإنها تعين القائم بحل مشكلة معينة على التوصل إلى جوهر المشكلة، كما أنها تساعد المهندس في مواجهة المحاجة المبنية على المغالطة. ويجري أيضًا توضيح كيف تسفر الطريقة عن نتائج مثمرة عندما يتم إدماجها مع أسلوب الأمن والسلامة المعروف باسم تحليل شجرة الأخطاء. تُستعمل الطريقة أيضًا في الاستنباط الاختياري وفي اتخاذ القرارات المعضدة بالمعرفة.