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Buckling Analysis of Steel Plates with Stiffened Openings Subjected to In-plane Combined Stresses

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Abstract. This paper presents a theoretical elastic buckling analysis of steel rectangular plates with stiffened or unstiffened openings. The plate is subjected to in-plane combined linearly varying compressive and shearing edge stresses. In this analysis, the plate is assumed simply supported along all edges. The location and the size of the opening, the stiffeners position and the cross section area of the stiffeners are the main parameters studied. Moreover, the compressive stress gradient (ψ) and the ratio of applied normal and shear stresses on the plate edges are taken into account. The minimum potential energy technique is used to analyze and solve the general mathematical formulations of the plate buckling under the aforesaid situation. Explicit equations as well as simplified ones are derived from which the buckling load of the plate can be obtained. The present predictions of the analysis are shown to be in a good agreement with the theoretical and experimental previous studies available. The predicted equations, graphs and the computer program are found to be more rational, accurate and versatile for design engineers.
***Keywords*:** Elastic Buckling, Steel Rectangular Plates, Stiffened or Unstiffened Opening, Compression, Shear, Minimum Potential Energy Technique, Fourier

1. Introduction

In certain design situations of steel structures, it is necessary to construct plates with openings. These plates may be part of webs or flanges of girders, columns or large sheet structures either stiffened or unstiffened. The introduction of an opening changes the stress distribution within the structural member and May, in many cases; change the member's mode of failure [1]. Numerous researchers focused on determining the compressive buckling stresses, for square or rectangular plates with a central circular opening or with a cut-out at any arbitrary position within the plate, due to uniform edge stresses [1, 9, 10, and 11]. Taking a further step, the author did an explicit form from which the buckling stresses of rectangular plates, subjected to linearly distributed edge compression only, can be calculated taking into consideration the presence of unstiffened openings in arbitrary locations within the plate [5]. As an extension to the research by the author of [5], Abdrabou and Nour [2] studied the effect of adding stiffeners to the opening, in [4] and [7] Maiorana et al, buckling analyses of square and rectangular plates with circular and rectangular holes in various positions subjected to axial compression and bending moment are studied.

In this paper, there are two main goals: (1) to develop a general method of analysis for finding the buckling loads for one of the main structural components which is the rectangular plate with openings located at any arbitrary position with or without stiffeners and subjected to combined in-plane linearly varying edge compressive and shearing loads, (2) to develop simple expressions, for common cases of loading and critical locations of stiffened or unstiffened openings, from which the buckling coefficient can be manually calculated. The most interesting parameters studied, in this research, and did not merely include in the previous researches are: (1) the position of stiffeners, which can take any angle (γ) around the opening, (2) the effect of additional in-plane shearing stress on the critical buckling stress in the presence of stiffened or unstiffened openings and [3] taking into consideration the external work done due to the presence of stiffeners and openings, which was neglected in the previous research by the author [5]. Moreover, complex mathematical solutions that lead to simple and explicit expressions to solve the problems of buckling in a more rational way are achieved. It has to be noticed that the influence of openings, with or without stiffeners, on the critical buckling stresses of plates is not treated in some codes [8 and 6].

2. Methodology and Research Strategy

The computation of the critical buckling load is based on the minimum potential energy technique. A rigorous analysis, using the exact solution of the differential equations of equilibrium, has been presented. Further, a computer program to achieve a parametric study is developed. Furthermore, the results of the present analysis have been verified by means of comparison with previous researches. Finally, design graphs and empirical equations for practical purposes are achieved.

3. Theoretical Approach

3.1 Buckling Analysis

All investigations in this study have been carried out with the help of the program developed in Appendix-I, which is constructed specially to solve the present differential equations and the required double integrations of the potential energy method. As shown in Fig. (1), the opening lies at any arbitrary position. The location of opening corner (o) has arbitrary coordinates (xo, yo) with respect to x- and y-axes. The stiffeners around the opening may have any arbitrary inclination angle (γ) varying from (0-45º). The plate is assumed to have simply supported edges, so that the double Fourier series can represent the deflection surface. To simplify the mathematical calculations, it was found that two terms of the double Fourier series are accurate enough. This function obeys the boundary conditions of the plate since each term of Eq. (1) vanishes when x = 0, y = 0, x = a and y = b and so do the second derivatives $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$.

$$w = C_1 \sin\alpha x \sin\beta y + C_2 \sin 2\alpha x \sin 2\beta y \tag{1}$$

where, $\alpha = \dfrac{\pi}{a}$, $\beta = \dfrac{\pi}{b}$ and (a , b) are the plate dimensions.

This deflection function can be expressed easily in two functions ϕ_1 and ϕ_2, which are functions in (x, y) and (2x, 2y), respectively. This simplifies on the calculations later and w becomes:

$$w = C_1\phi_1 + C_2\phi_2 \tag{2}$$

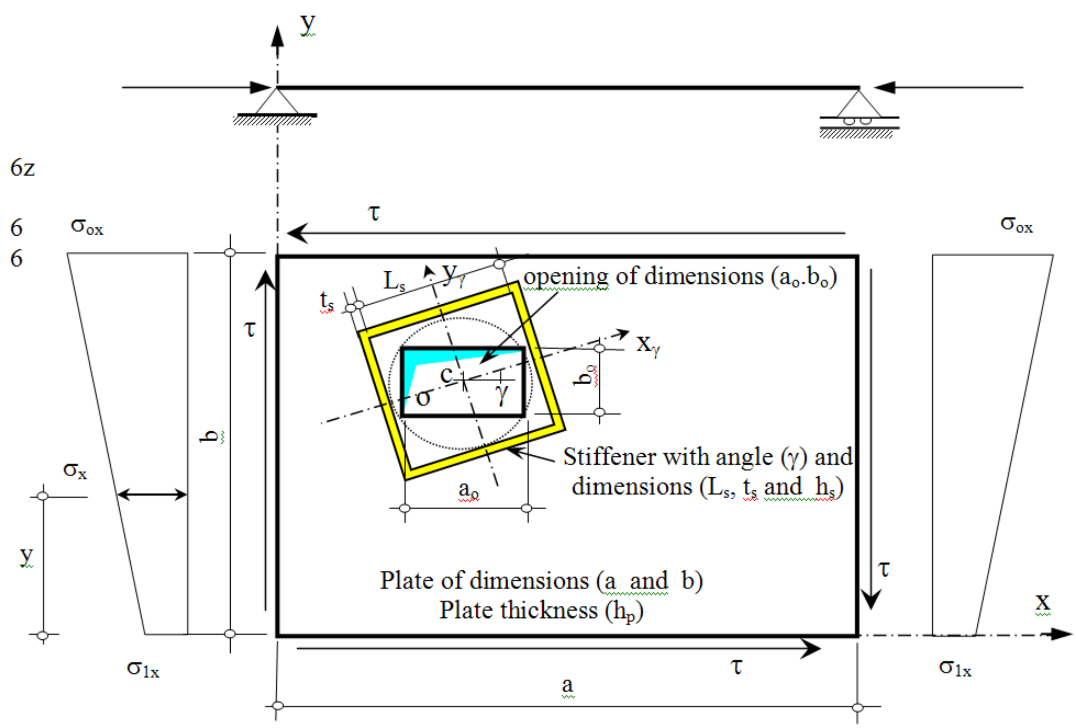

Fig. (1). Geometry and loading for the general case of rectangular plate with opening

The common general expression of strain energy stored (U_i) in a system deflected in this form is [1, 2, 3, 6]:

$$U_i = \frac{1}{2} D_i \iint_{area} \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\mu) \left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy \tag{3}$$

The general total work done by the linearly varying compressive and shearing edge loads (T_i), as shown in Fig.(1) is as follows:

$$T_i = \frac{1}{2} \iint_{area} \sigma_x h_p \left(\frac{\partial w}{\partial x} \right)^2 dxdy + \iint_{area} \tau_{xy} h_p \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dxdy \tag{4}$$

where:

1. $\sigma_x = \sigma_{ox} \left[\Psi' + \dfrac{y}{b}(1-\Psi') \right]$,
2. the plate, stiffeners and opening are denoted by (p, s and o) instead of (i); see Eqs. (3, 4, 6 and 7) and
3. the integration over the area can be for the plate, the stiffeners or the opening in Eqs. (3, 4, 6 and 7).

To achieve the partial derivatives, four functions have been defined in the following forms:

$$\phi_j = \phi_j(x,y) = \sin(j\alpha x)\sin(j\beta y),\ j = 1,2 \tag{5a}$$

$$\varphi_j = \varphi_j(x,y) = \cos(j\alpha x)\cos(j\beta y),\ j = 1,2 \tag{5b}$$

$$\chi_j = \chi_j(x,y) = \cos(j\alpha x)\sin(j\beta y),\ j = 1,2 \tag{5c}$$

$$\bar{\chi}_j = \bar{\chi}_j(x,y) = \sin(j\alpha x)\cos(j\beta y),\ j = 1,2 \tag{5d}$$

Substituting from Eqs. (2, 5a, 5b, 5c and 5d) in Eqs. (3 and 4), the strain energy stored (Ui) and the total work done (Ti) can be rewritten as follows:

$$U_i = D_i \iint_{A_i} \Big\{ \lambda_1 \left[C_1^2 \phi_1^2 + 8C_1C_2\phi_1\phi_2 + 16C_2^2\phi_2^2 \right] + \\ + \lambda_2 \left[C_1^2\left(\varphi_1^2 - \phi_1^2\right) + 8C_1C_2\left(\varphi_1\varphi_2 - \phi_1\phi_2\right) + 16C_2^2\left(\varphi_2^2 - \phi_2^2\right) \right] \Big\} dxdy, \tag{6}$$

$$T_i = \iint_{A_i} \Big\{ \lambda_{3i} \left[\Psi' + \frac{1-\Psi'}{b} y \right] \left[C_1^2\chi_1^2 + 4C_1C_2\chi_1\chi_2 + 4C_2^2\chi_2^2 \right] + \\ + \lambda_{4i} \left[C_1^2\chi_1\bar{\chi}_1 + 2C_1C_2\left(\chi_1\bar{\chi}_2 + \bar{\chi}_1\chi_2\right) + 4C_2^2\chi_2\bar{\chi}_2 \right] \Big\} dxdy \tag{7}$$

where, Di can be the plate stiffness (Dp) or the stiffness of the stiffener (Ds),

$$\lambda_1 = 0.5\left(\alpha^2 + \beta^2\right)^2,$$

$$\lambda_2 = 2(1-\mu)\alpha^2\beta^2,$$

$$\lambda_{3i} = 0.5\sigma_{ox}\alpha^2 h_i$$

$$\lambda_{4i} = F_{ts}\sigma_{ox}\alpha\beta h_i \text{ and}$$

$$F_{ts} = \text{shear ratio} = \frac{\tau_{xy}}{\sigma_{ox}}.$$

In Appendix-I the definition of the previous integrals is tabulated. As a matter of fact, the total strain energy (Ut) or the total work done (Tt) of a plate with a stiffened opening should be modified to cope with the existence of the stiffeners and opening. Thus, the total strain energy (Ut) becomes:

$$U_t = U_p + U_s - U_0 \tag{8}$$

where,

$$U_p = D_p\left[0.25\lambda_1 ab\left(C_1^2 + 16C_2^2\right)\right],$$

$$U_s = C_1^2\left[D_s\lambda_1 I_s\phi_1^2 + D_s\lambda_2 I_s\left(\varphi_1^2 - \phi_1^2\right)\right] + 8C_1C_2\left[D_s\lambda_1 I_s\phi_1\phi_2 + D_s\lambda_2 I_s\left(\varphi_1\varphi_2 - \phi_1\phi_2\right)\right] + \\ + 16C_2^2\left[D_s\lambda_1 I_s\phi_2^2 + D_s\lambda_2 I_s\left(\varphi_2^2 - \phi_2^2\right)\right], \text{ and}$$

$$U_o = C_1^2\left[D_p\lambda_1 I_o\phi_1^2 + D_p\lambda_2 I_o\left(\varphi_1^2 - \phi_1^2\right)\right] + 8C_1C_2\left[D_p\lambda_1 I_o\phi_1\phi_2 + D_p\lambda_2 I_o\left(\varphi_1\varphi_2 - \phi_1\phi_2\right)\right] + \\ + 16C_2^2\left[D_p\lambda_1 I_o\phi_2^2 + D_p\lambda_2 I_o\left(\varphi_2^2 - \phi_2^2\right)\right]$$

The total work done (Tt) is expressed as follows:

$$T_t = T_p + T_s - T_o \tag{9}$$

where,

$$T_p = \left[0.125ab\lambda_{3p}(1+\Psi')\left(C_1^2+4C_2^2\right)\right],$$

$$T_s = C_1^2\left[\lambda_{3s}I_s\chi_1^2+\lambda_{4s}I_s\chi_1\bar{\chi}_1\right]+2C_1C_2\left[2\lambda_{3s}I_s\chi_1\chi_2+\lambda_{4s}I_s\left(\chi_1\bar{\chi}_2-\bar{\chi}_1\chi_2\right)\right]+ + 4C_2^2\left[\lambda_{3s}I_s\chi_2^2+\lambda_{4s}I_s\chi_2\bar{\chi}_2\right],$$

and

$$T_o = C_1^2\left[\lambda_{3p}I_o\chi_1^2+\lambda_{4p}I_o\chi_1\bar{\chi}_1\right]+2C_1C_2\left[2\lambda_{3p}I_o\chi_1\chi_2+\lambda_{4p}I_o\left(\chi_1\bar{\chi}_2-\bar{\chi}_1\chi_2\right)\right]+ + 4C_2^2\left[\lambda_{3p}I_o\chi_2^2+\lambda_{4p}I_o\chi_2\bar{\chi}_2\right]$$

See Appendix –I for the detail of the previous integrations for Ip, Io and Is. From Eqns. (8 and 9), the total potential energy (V) of the plate with stiffened opening can be expressed in a general form as follows:

$$V = U_t - T_t \tag{10}$$

Based on the minimum potential energy technique, the stress σox can be evaluated by partial differentiation of total potential energy (V) with respect to unknown coefficients C1 and C2:

$$\frac{\partial \mathrm{V}}{\partial \mathrm{C}_1} = 0 \tag{11a}$$

$$\frac{\partial \mathrm{V}}{\partial \mathrm{C}_2} = 0 \tag{11b}$$

This leads to:

$$2a_{11}C_1 + a_{12}C_2 = 0 \tag{12}$$

$$a_{12}C_1 + 2a_{22}C_2 = 0 \tag{13}$$

For non-trivial solution:

$$a_{12} = 2\sqrt{a_{11}a_{22}} \tag{14}$$

The coefficients a_{11}, a_{12} and a_{22} can rewritten as:

$a_{11} = Z_1 + \sigma_{ox}Z_2$,

$a_{12} = Z_3 + \sigma_{ox}Z_4$, and

$a_{22} = Z_5 + \sigma_{ox}Z_6$, then,

$$\sigma_{ox}^2\left(4Z_2Z_6 - Z_4^2\right) + \sigma_{ox}\left(4Z_1Z_6 + 4Z_2Z_5 - 24Z_3Z_4\right) + \left(4Z_1Z_5 - Z_3^2\right) = 0 \tag{15}$$

$$\sigma_{ox,cr} = \frac{1}{2f_1}\left(-f_2 \pm \sqrt{f_2^2 - 4f_1f_3}\right)$$

where,

$f_1 = \left(4Z_2Z_6 - Z_4^2\right)$,

$f_2 = \left(4Z_1Z_6 + 4Z_2Z_5 - 2Z_3Z_4\right)$,

$f_3 = \left(4Z_1Z_5 - Z_3^2\right)$,

and the values of Z_1, Z_2, Z_3, Z_4, Z_5 and Z_6 are as defined in Appendix-I. The buckling coefficient for a rectangular plate with stiffened opening at any arbitrary position (modified buckling coefficient = K_m) can be expressed as follows:

$$K_m = \sigma_{ox,cr}\frac{12\left(1-\mu^2\right)}{\pi^2 E}\left(\frac{b}{h_p}\right)^2 \tag{16}$$

3.2 Verification of the Predicted Expressions and the Computer Program

The author's developed computer program is based on the mathematical derivations in order to solve the differential equations and the double integrations of the surface area of the plate. To verify the results of the present analysis different applications have been presented.

Fig. (2) shows the relationship between the buckling stress ratio ($\sigma_{cr,\ opening}/(\sigma_{cr,\ solid})$ and the ratio of the opening side to the plate side (a_o/a) for square plates with square openings. From this figure, it can be distinctly seen that the comparison between the results of the present analysis and the previous theoretical and experimental results gives good agreement. Also, Table (1) shows close scrutiny of selection of experimental results [2] and results out of the present predicted analysis. In this table, the differences between the results of the experimental work of reference [2] and the results of the present predicted analysis varied between (0.17 % and 1.27 %). Moreover, the presence of the diagonal stiffeners (γ = 45^o) around a rectangular opening in a rectangular plate has been also verified. In this verification, good agreement between the value of the critical buckling load (5.7 ton) of an example solved in the Appendix of reference [2] and that given by the present analysis (5.73 ton). The comparisons between the present predictions and the previous researches, whether experimental or theoretical, reflect the accuracy of the present results.

Table (1). Selective cases for uniform compressive stresses (ψ =1.0).

Beam No.	**Thickness of plate [mm]**	**Opening size [mm]**	**Stiffener type**	**Experimental critical load [2]**	**Present theoretical load**	***Difference***
				Pcr,exp [t]	**Pcr,th [t]**	
B1	6	No opening	No stiffener	17.2	17.23	0.17 %
B2	4	250 x 200	No stiffener	4.60	4.737	2.98 %
B3	4	250 x 200	Stiffeners with (γ = 0^o)	5.50	5.57	1.27 %

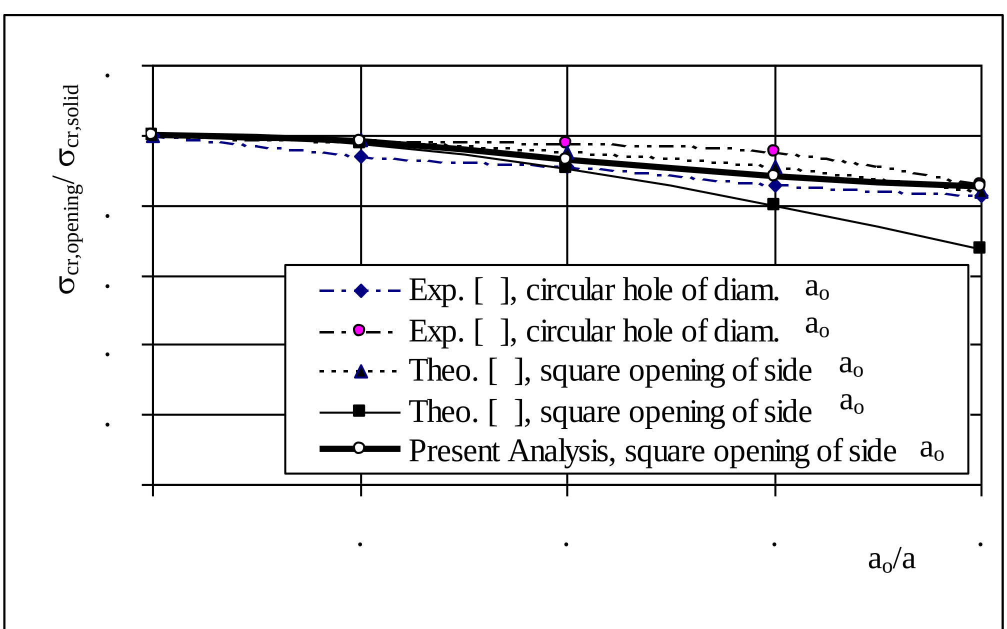

Fig. (2). Comparisons of critical buckling stresses for square plates with square openings.

4. Empirical Equations and Design Graphs

In order to utilize the predicted analysis, the results of the experimental tests and previous analyses have first been verified. Then by using the present analysis, the most common cases, in practice for square or rectangular plates with stiffened or unstiffened openings in which the plates are subjected to in-plane uniform or linearly varying compression combined with shearing force, are theoretically performed. A computer program is specially developed by the authors to solve the previous mathematical equations. Different cases of applied stresses and geometries of plates can be analyzed using this program. Hereafter, design graphs and empirical equations are also proposed from which the buckling coefficient can be directly calculated for a wide range of common cases. These graphs and predicted empirical equations, when compared with the present general equation (16), are found to be accurate and versatile ones.

4.1 Influence of Stiffeners Height and Location Around the Opening

Figures (3a , 3b and 3c) show the relationships between the ratios of the heights of stiffeners and the plates thickness (h_s/h_p) and the modified buckling coefficients (K_m). These relationships have been introduced for rectangular plates of aspect ratios ($\phi = a/b = 1, 1.2$ and 1.4), aspect ratios of central opening (1, 1.2 and 1.4) and stiffeners of inner lengths equal to the diameter of a circle tangent to the opening corners ($L_s = \sqrt{a_o^2 + b_o^2}$) and inclined with angle ($\gamma = 0.0^o$ and 45^o). Also, the stiffener thickness is proposed to be more or equal to the plate thickness. It can be emphasized that the best ratio of stiffener height to the plate thickness (h_s/h_p / 12) at which the value of the modified buckling coefficient (K_m) is more or equal to the corresponding value of buckling coefficient (K) for a solid plate (without opening).

4.2 Plates With Stiffened and Unstiffened Opening Subjected to the Combined Action of Linearly Varying Compression and Shear

Fig. (4) for unstiffened openings as well as Figs. (8, 9 and 10) for stiffened opening are considered as design graphs to obtain directly the buckling coefficient of square plates with central square opening for different gradients of compression loads ($\psi = \sigma_{1x}/\sigma_{ox} = 1.0, 0.5$ and 0.0) and for various shear ratios (τ/σ_{ox}). The opening sizes provided in these graphs are denoted by the ratio between the opening side and the plate side (a_o/a). It can be concluded that the increase of the shear ratio has a considerable effect in reducing the value of the buckling coefficient as well as the increase of the opening area. Form Figs. (5a, 5b and 5c), it can be found that the buckling coefficient ratios ($K_m/(K_m)_{\tau=0}$) for different opening sizes ($a_o/a = 0.30$) are merely affected by the shear ratio only. Empirical equations for different stress gradient have been proposed in the sense that the lower bound of the relationships between ($K_m/(K_m)_{\tau=0}$) and (a_o/a) is fitted.

The ratio $\left[K_m/(K_m)_{\tau=0}\right]$ for square plates with stiffened and unstiffened central square openings in which the plates are subjected to the combined action of linearly varying compressive and shear loads can be obtained from the following proposed empirical equations as follows:

For ψ= 1.0:

$$\frac{K_m}{(K_m)_{\tau=0}} = 1.0 - 0.108\left(\frac{\sigma_{ox}}{\tau}\right) \tag{17}$$

For ψ= 0.5:

$$\frac{K_m}{(K_m)_{\tau=0}} = 1.0 - 0.17\left(\frac{\sigma_{ox}}{\tau}\right) \tag{18}$$

For ψ= 0.0:

$$\frac{K_m}{(K_m)_{\tau=0}} = 1.0 - 0.30\left(\frac{\sigma_{ox}}{\tau}\right) \tag{19}$$

The values of the modified buckling coefficient at zero shear $(K_m)_{\tau=0}$ in equations (17, 18 and 19) can be calculated as follows:

In case of unstiffened openings: From Fig. (6),

$$(K_m)_{\tau=0} = K_{s,\psi}\left[1.0 + 0.044\left(\frac{a_o}{a}\right) - 3.0\left(\frac{a_o}{a}\right)^2 + 5.0\left(\frac{a_o}{a}\right)^3\right] \tag{20}$$

In case of stiffened openings: From Fig. (7),
For stiffeners with angle $\gamma = 45^o$

$$\left(K_m\right)_{\tau=0} = K_{s,\psi}\left[1.0+0.67\left(\frac{a_o}{a}\right)-4.25\left(\frac{a_o}{a}\right)^2+7.92\left(\frac{a_o}{a}\right)^3\right], \tag{21a}$$

and for stiffeners with angle $\gamma = 0^o$

$$\left(K_m\right)_{\tau=0} = K_{s,\psi}\left[1.0+6.78\left(\frac{a_o}{a}\right)-43.875\left(\frac{a_o}{a}\right)^2+72.92\left(\frac{a_o}{a}\right)^3\right], \tag{21b}$$

In the previous equations (20, 21a and 21b), the values of the buckling coefficient ($K_{s,\psi}$) of the solid plates can be taken form literature according to the values of stress gradient (ψ) or as follows:

$K_{s,\psi}$ = 4.0, 5.33 and 8.0 for ψ = 1.0, 0.5 and 0.0, respectively.

5. Summary and Conclusions

Good results from all previous graphs and attached empirical equations are clearly achieved. In this respect, present proposed equations Eq. (17) to Eq. (21b) or the present graphs Fig. (3) to Fig, (10) can be easily applied to obtain the buckling coefficient for square plates with central square stiffened or unstiffened openings. These equations are just a sample of a particular case of square plate and many other equations for different plate rectangularity can be easily developed using the authors' program. Moreover the computer program can be directly used to estimate the buckling coefficient for different variables as: the plate rectangularity ratio ϕ, the plate thickness (h_p), the size of the opening (a_o and b_o), the arbitrary location of the opening within the plate, the stiffeners height, the stiffener thickness, the location of stiffeners around the opening (γ^o), the stress gradient (ψ) and the shear ratio (τ/σ_{ox}).

From the present theoretical investigation the following conclusions are drawn:

The present mathematical formulation is straightforward, high accuracy is achieved with previous theoretical and experimental results.

With the present program, the effect of various parameters on the buckling coefficient can be accurately yet simply studied; hence eliminating any time consuming process as finite element or other numerical methods.

From the given results, it can be seen that the effect of introducing unstiffened openings on the buckling stresses of plates is significant and the plate should be carefully treated.

Samples of design graphs as well as simplified design empirical equations have been proposed to predict the buckling coefficient for square plates provided by stiffened or unstiffened openings in which the plate is subjected to combined in-plane linearly varying compressive and shearing stresses.

Important assumptions such as the size of the stiffeners (h_s/h_p / 12 and t_s/h_p /1.0 and $L_s = \sqrt{a_o^2 + b_o^2}$) and the position of stiffeners ($\gamma = 0^o$) around the opening to avoid any reduction of the critical buckling load due to the presence of openings are found to be suitable recommendation for design purposes.

6. References

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Appendix-I

The integral operator in two dimensions is defined as: $I_i f\left(x,y\right)=\int_{x_l}^{x_u}\int_{y_l}^{y_u} f(x,y)dxdy$

And (i) take the symbols (p, o, and s) for the plat, opening and stiffener, respectively.
Table of required integrals on the area of plate (A_p), opening (A_o) and stiffeners (A_s):

Number	Functions	A_p	A_o	A_s
1	$(\phi_1)^2$	$I_p(\phi_1)^2$	$I_o(\phi_1)^2$	$I_s(\phi_1)^2$
2	$(\phi_2)^2$	$I_p(\phi_2)^2$	$I_o(\phi_2)^2$	$I_s(\phi_2)^2$
3	$\phi_1\phi_2$	$I_p(\phi_1\phi_2)$	$I_o(\phi_1\phi_2)$	$I_s(\phi_1\phi_2)$
4	$(\varphi_1)^2$	$I_p(\varphi_1)^2$	$I_o(\varphi_1)^2$	$I_s(\varphi_1)^2$
5	$(\varphi_2)^2$	$I_p(\varphi_2)^2$	$I_o(\varphi_2)^2$	$I_s(\varphi_2)^2$
6	$(\varphi_1\varphi_2)$	$I_p\,(\varphi_1\varphi_2)$	$I_o(\varphi_1\varphi_2)$	$I_s(\varphi_1\varphi_2)$
7	$(\chi_1)^2$	$I_p(\chi_1)^2$	$I_o(\chi_1)^2$	$I_s(\chi_1)^2$
8	$(\chi_2)^2$	$I_p(\chi_2)^2$	$I_o(\chi_2)^2$	$I_s(\chi_2)^2$
9	$(\chi_1\chi_2)$	$I_p(\chi_1\chi_2)$	$I_o(\chi_1\chi_2)$	$I_s(\chi_1\chi_2)$
10	$(\chi_1\chi_1)$	$I_p(\chi_1\chi_1)$	$I_o(\chi_1\chi_1)$	$I_s(\chi_1\chi_1)$
11	$(\chi_2\chi_2)$	$I_p(\chi_2\chi_2)$	$I_o(\chi_2\chi_2)$	$I_s(\chi_2\chi_2)$

The values of constants are:

$$z_1=\lambda_1\left[0.25D_pab+D_sI_s\phi_1^2-D_0I_0\phi_1^2\right]+\lambda_2\left[D_sI_s\left(\varphi_1^2-\phi_1^2\right)-D_pI_0\left(\varphi_1^2-\phi_1^2\right)\right],$$

$$z_2=-\left[0.125ab\lambda_{3p}\left(1+\overline{\psi}\right)+\lambda_{3s}I_s\chi_1^2-\lambda_{3p}I_0\chi_1^2\right]-\left[\lambda_{4s}I_s\chi_1\overline{\chi}_1-\lambda_{4p}I_0\chi_1\overline{\chi}_1\right],$$

$$z_3=8\lambda_1\left[D_sI_s\phi_1\phi_2-D_pI_0\phi_1\phi_2\right]+8\lambda_2\left[D_sI_s\left(\varphi_1\varphi_2-\phi_1\phi_2\right)-D_pI_o\left(\varphi_1\varphi_2-\phi_1\phi_2\right)\right],$$

$$z_4=-4\left[\lambda_{3s}I_s\chi_1\chi_2-\lambda_{3p}I_0\chi_1\chi_2\right]-2\left[\lambda_{4s}I_s\left(\chi_1\overline{\chi}_2+\overline{\chi}_1\chi_2\right)-\lambda_{4p}I_0\left(\chi_1\overline{\chi}_2+\overline{\chi}_1\chi_2\right)\right],$$

$$z_5=16\lambda_1\left[0.25D_pab+D_sI_s\phi_2^2-D_pI_0\phi_2^2\right]+16\lambda_2\left[D_sI_s\left(\varphi_2^2-\phi_2^2\right)-D_pI_0\left(\varphi_2^2-\phi_2^2\right)\right],$$

$$z_6=-4\left[0.125ab\lambda_{3p}\left(1+\overline{\psi}\right)+\lambda_{3s}I_s\chi_2^2-\lambda_{3p}I_0\chi_2^2\right]-4\left[\lambda_{4s}I_s\chi_2\overline{\chi}_2-\lambda_{4p}I_0\chi_2\overline{\chi}_2\right].$$

Appendix-II

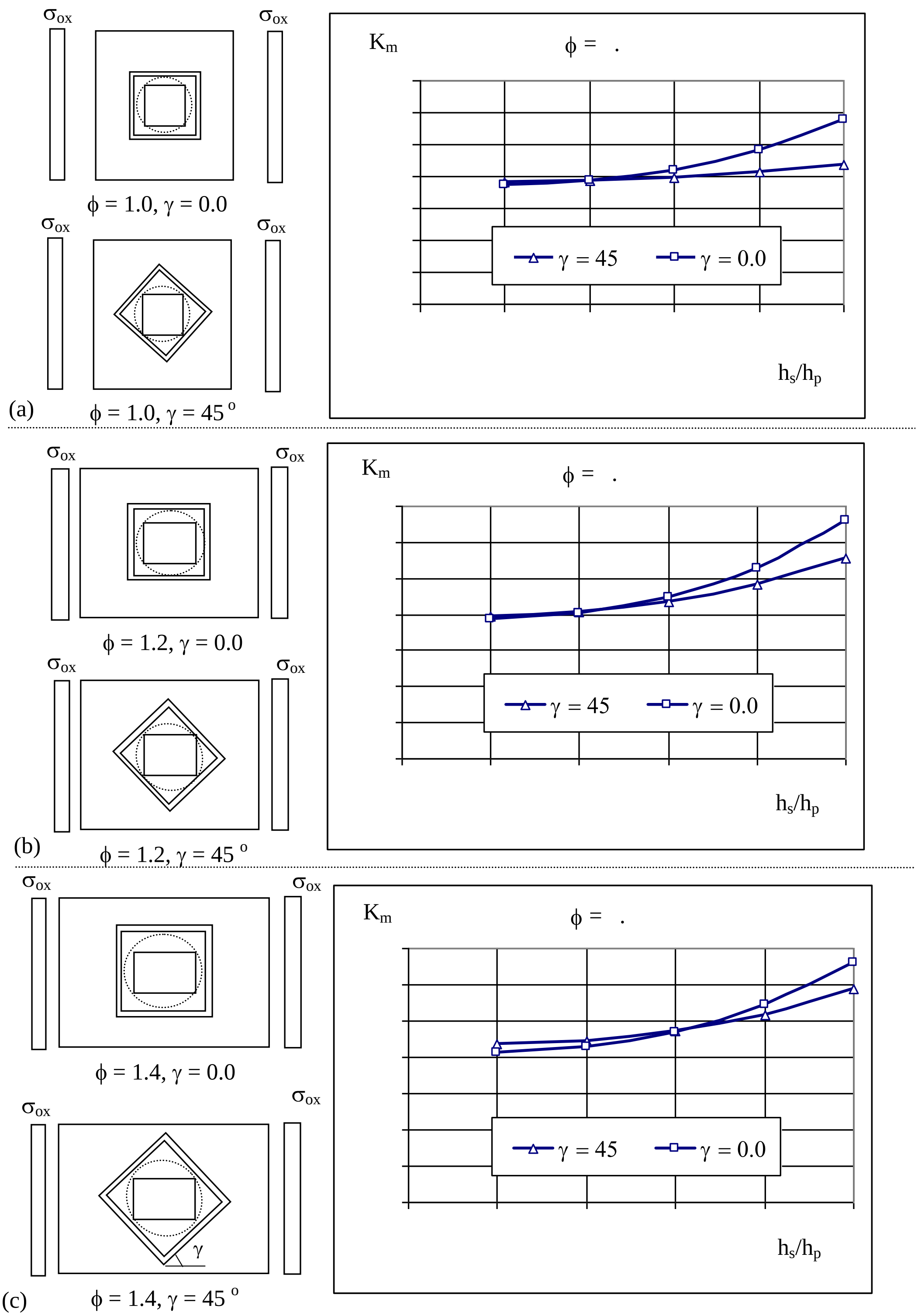


Fig. (3). Influence of the stiffeners height to the plate thickness ratio (hs/ hp) on the buckling coefficient (Km) for different plate aspect ratio (ϕ).

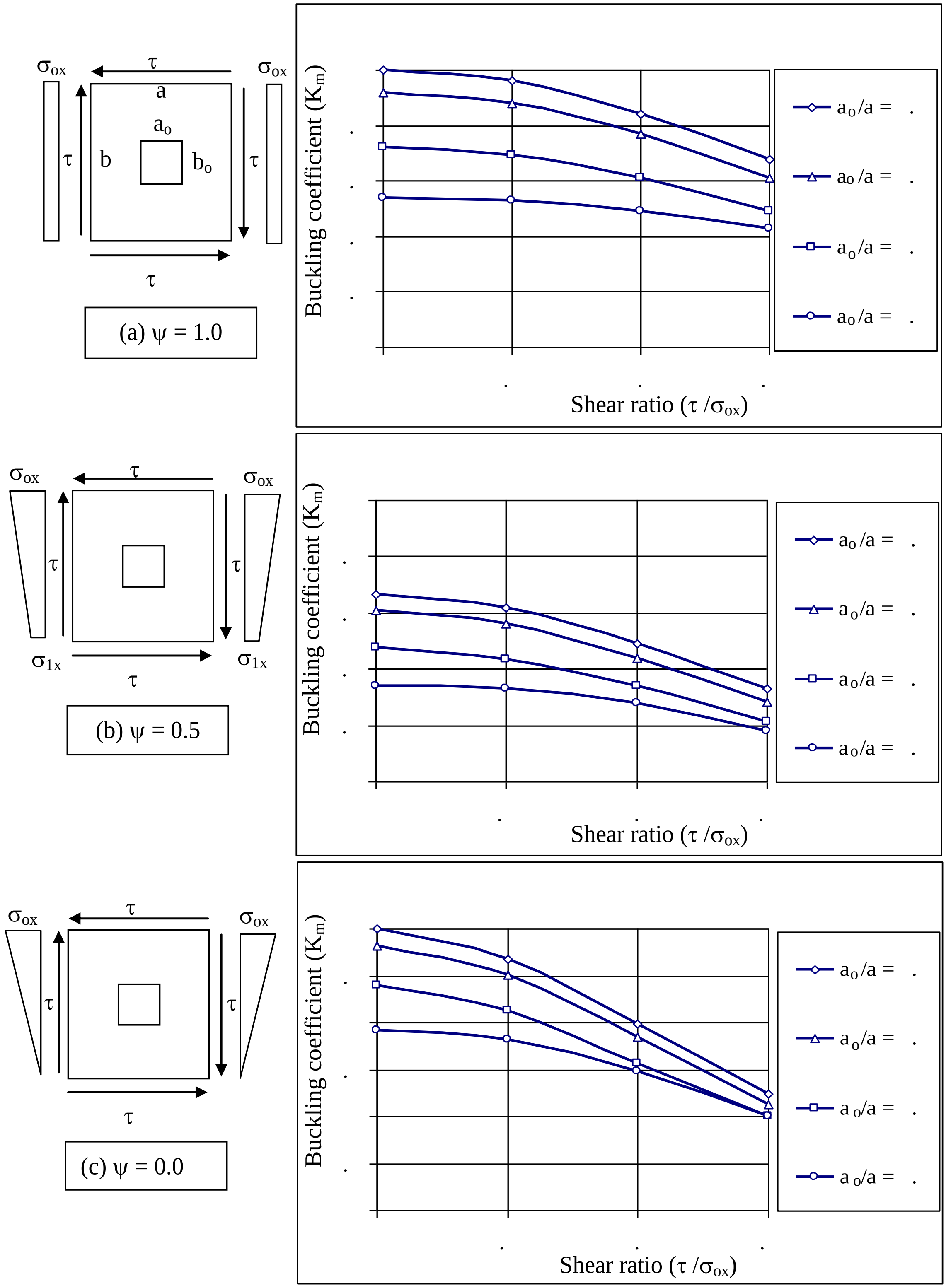


Fig. (4). Design curves for square plates (ϕ = a/b = ao/bo = 1.0) with unstiffened central square opening of different sizes under combined in-plane loading.

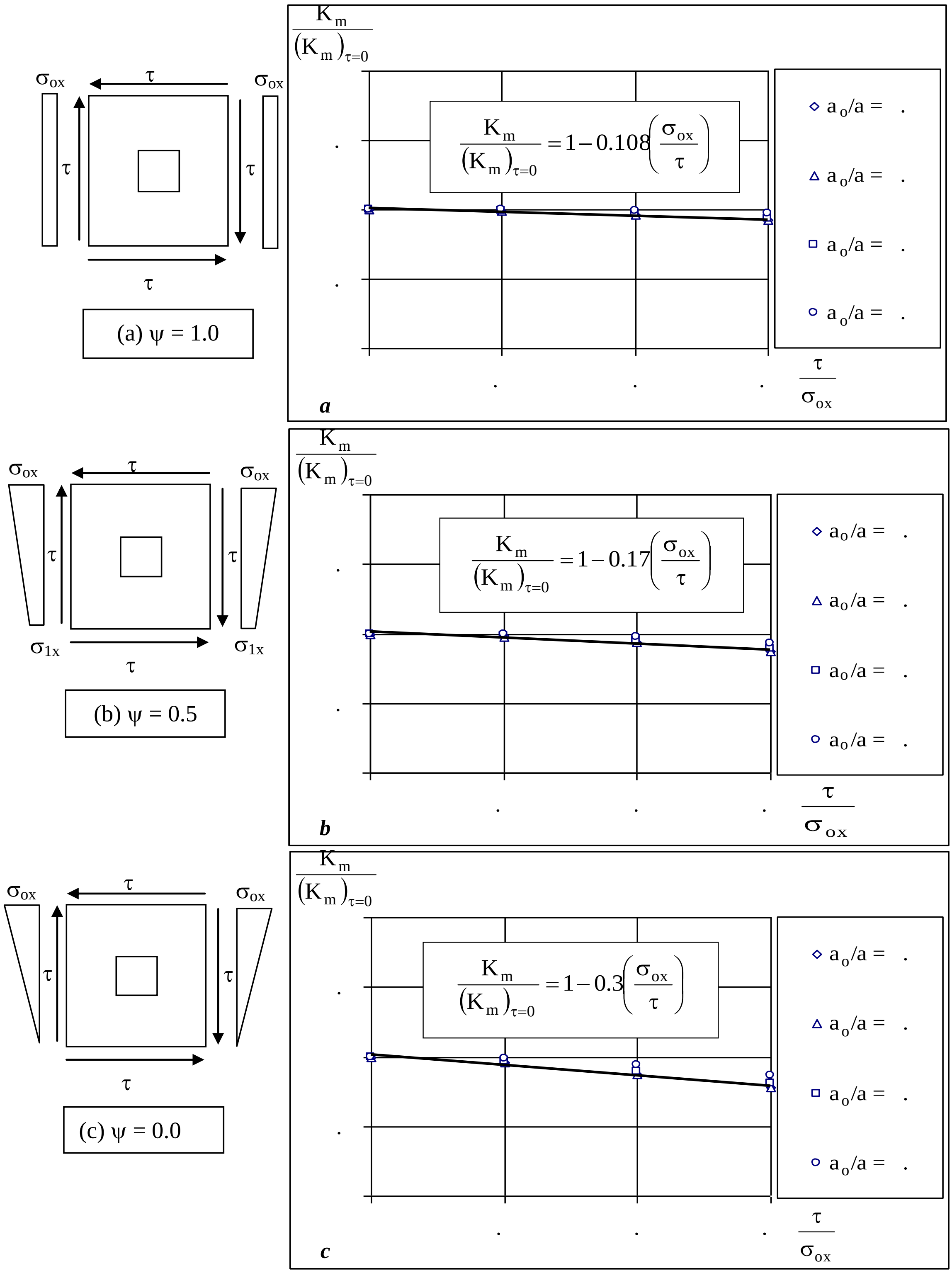


Fig. (5). Design formulas for square plates (ϕ = a/b = ao/bo = 1.0) with stiffened and unstiffened central square opening of different sizes under combined in-plane loading.

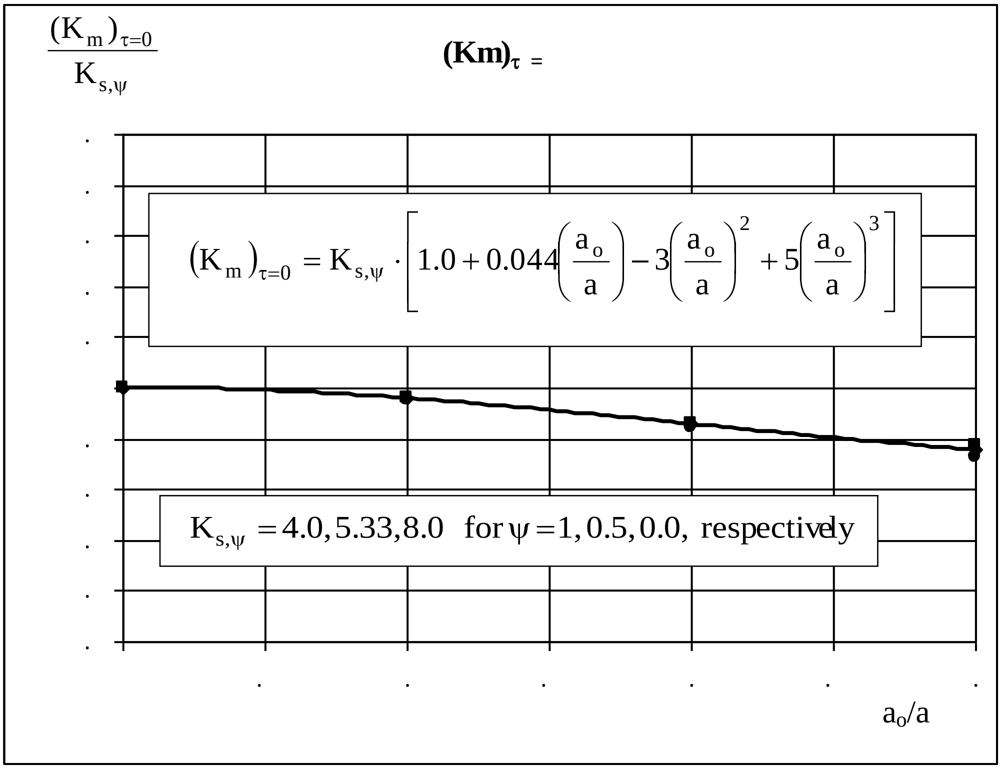


Fig. (6). Buckling coefficient for square plate with unstiffened opening under linearly varying compression.

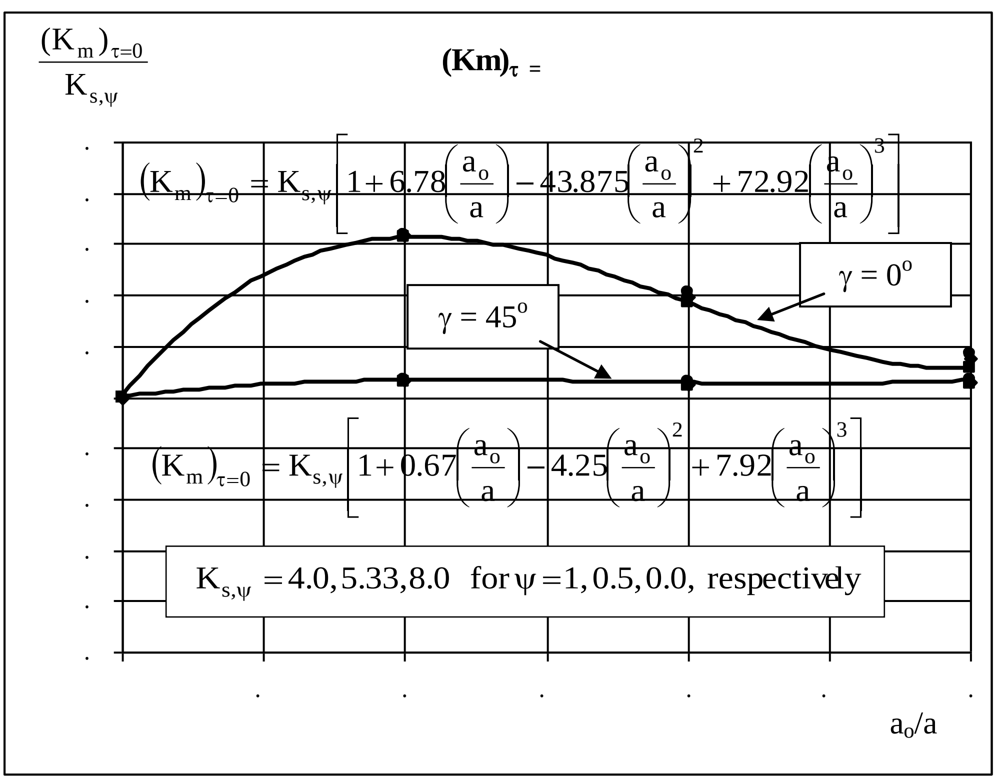
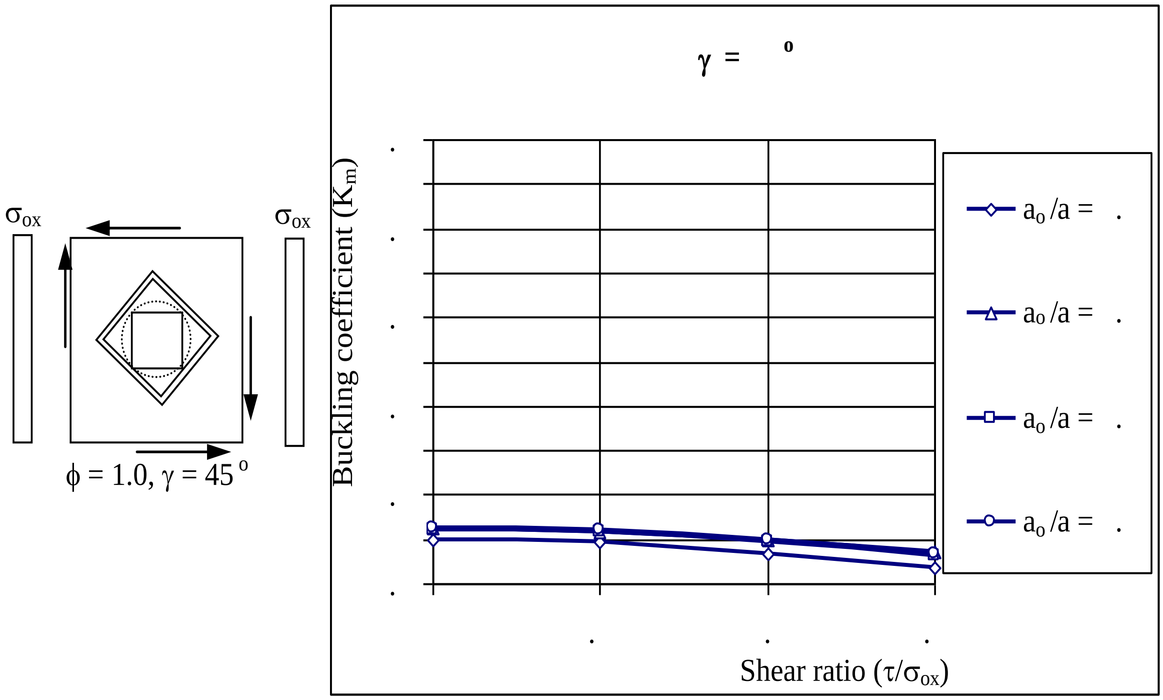
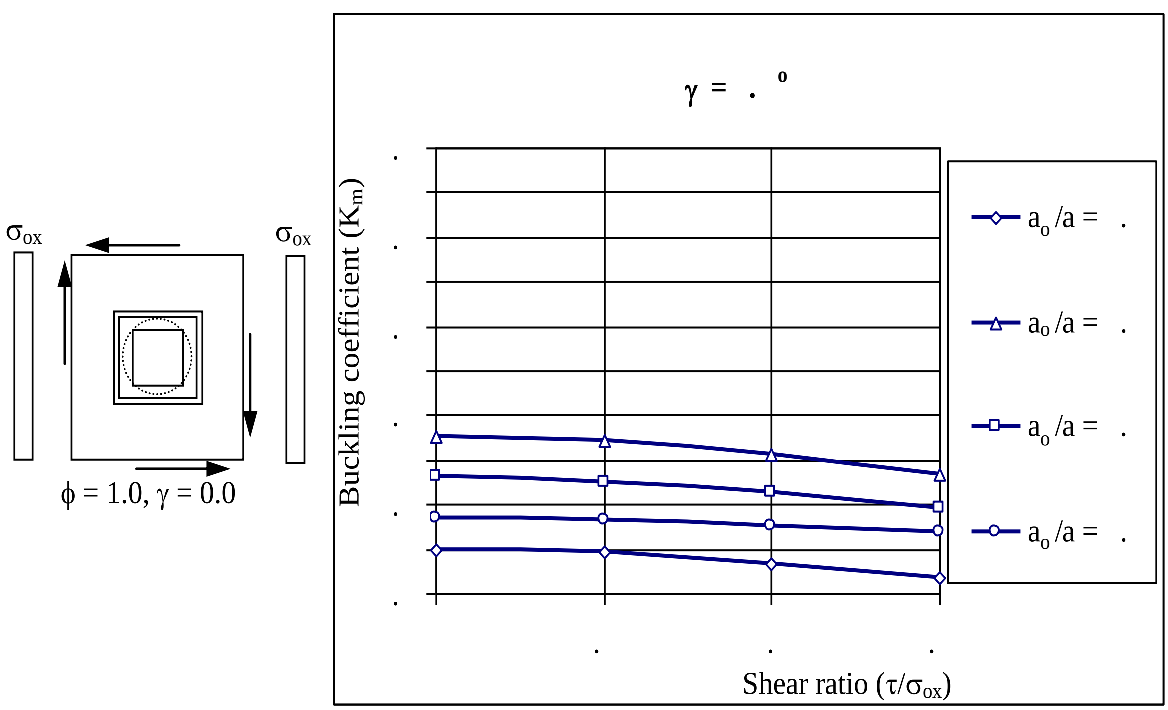


Fig. (7). Buckling coefficient for square plate with stiffened opening under linearly varying compression.



(a)



(b)

Fig. (8). Design curves for square plate with stiffened central square opening of different size subjected to stress gradient (ψ = 1.0).

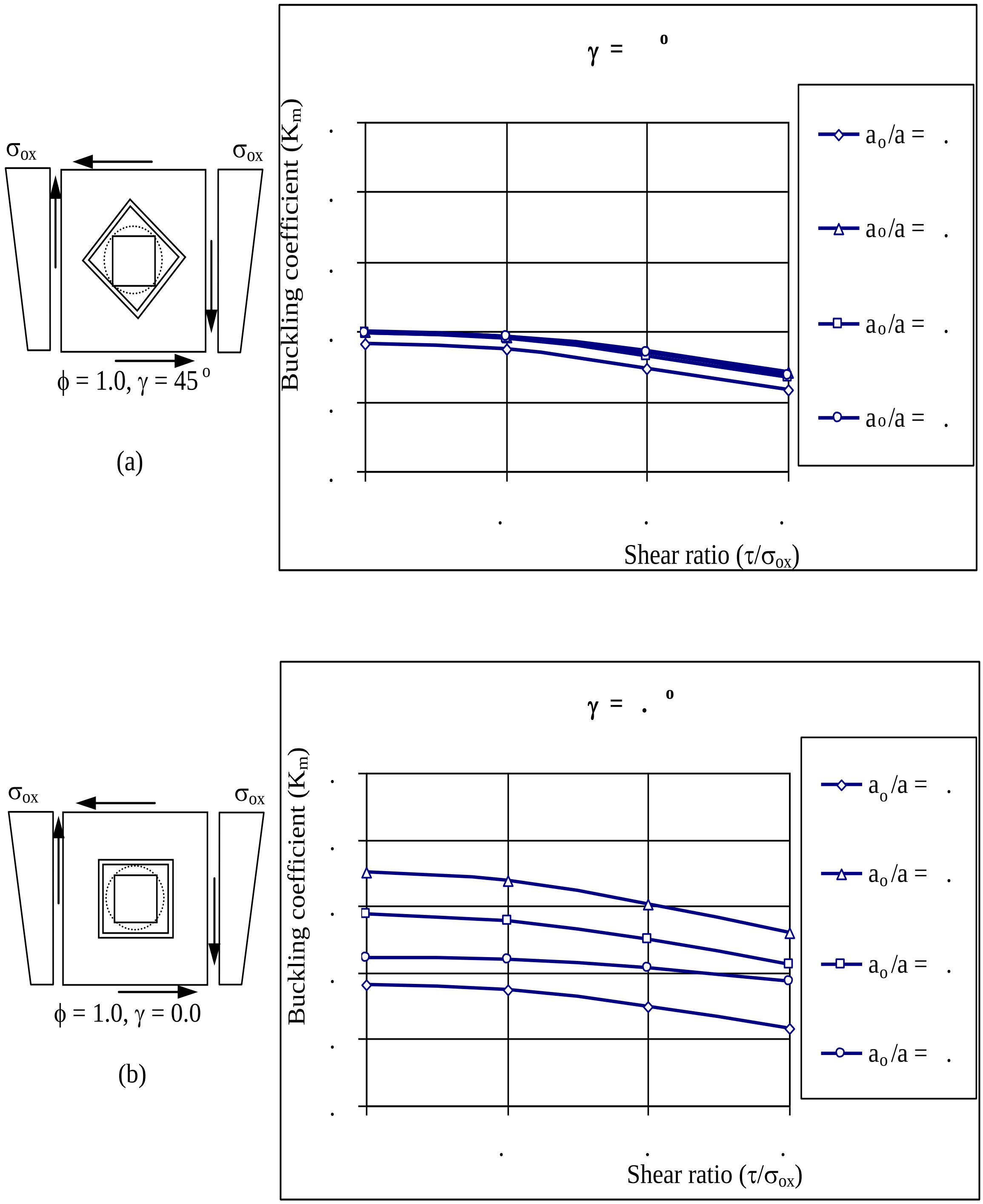


Fig. (9). Design curves for square plate with stiffened central square opening of different size subjected to stress gradient (ψ= 0.5).

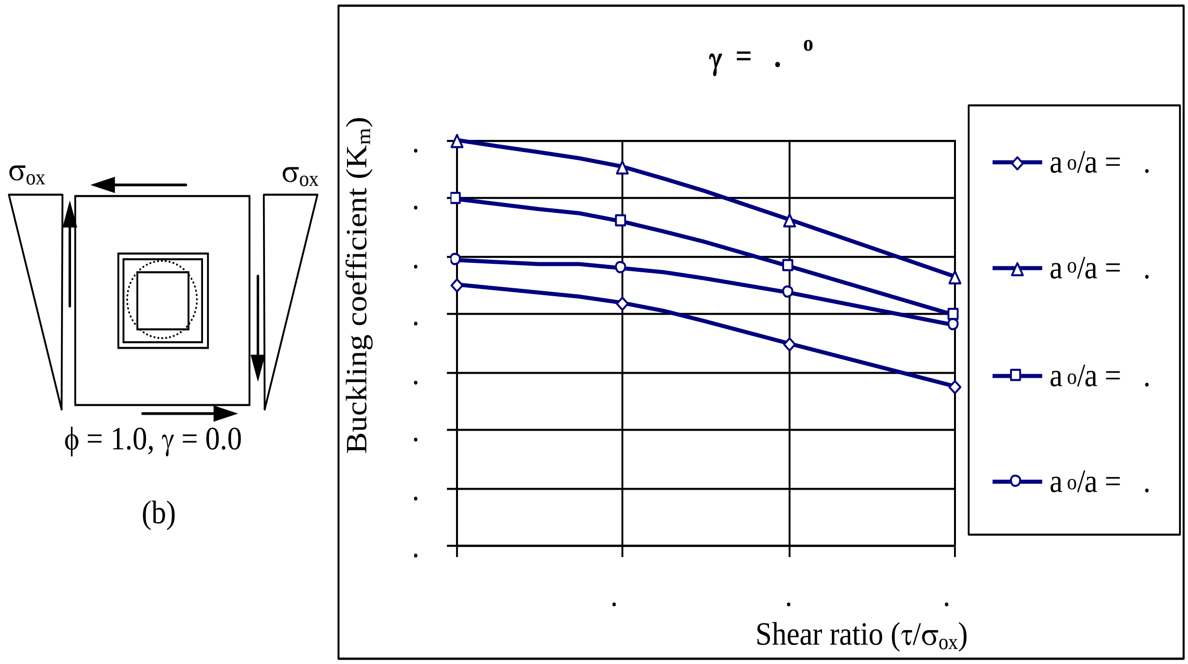


Fig. (10). Design curves for square plate with stiffened central square opening of different size subjected to stress gradient (ψ = 0.0)

دراسة الانبعاج للألواح الحديدية ذات الفتحات المقواة والمعرضة لأحمال مركبة في مستوي اللوح

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ملخص البحث. في هذا البحث تمت الدراسة النظرية التحليلية للانبعاج المرن للألواح المستطيلة الشكل ذات الفتحات المقواة أو غير المقواة والمعرضة لإجهادات في نفس مستوي اللوح (إجهادات الضغط الخطي المتغير أو المنتظم مع إجهادات القص). ومن أهم المتغيرات التي تم دراستها و التعامل معها هي: ١- مكان الفتحة في اللوح، ٢- أبعاد الفتحة، ٣- أوضاع التقويات حول الفتحة، ٤- أبعاد التقويات حول الفتحة. هذا بالإضافة إلي متغيرات التحميل و هي تشمل شكل إجهادات الضغط المتغير خطياً علي الحرف الخارجي للوح والمميز بدرجة الإجهاد (ψ)، وكذلك نسبة القص إلي الضغط، وللقيام بهذه الدراسة استخدمت طريقة "طاقة الوضع الدنيا" للحصول علي قيمة معامل الانبعاج المطلوب لأي من حالات الدراسة، وبناءً على ذلك تم استنتاج المعادلة العامة للإجهاد الحرج والتي تحتوي على كل المتغيرات السابقة، وأمكن أيضاً استنتاج معادلات أخري مبسطة تشتمل علي الحالات العملية المتعارف عليها ودون الحاجة إلى استخدام برنامج الكمبيوتر المرفق، وقد أظهرت نتائج المقارنة لحالات مختارة من الأبحاث السابقة توافقاً جيداً مع نتائج الدراسة الحالية، كما أظهرت سهولة التطبيق للمعادلات المستنتجة، وكذلك المنحنيات المرفقة.