Discrete-Time Dynamic Programming-Based and Robust LMI-Based Output-Feedback Controllers Design for an Induction Motor Speed Control

Ahmed Bensenouci
Electrical Engineering Department, Qassim University,
P.O. Box 6677, Buraydah 51452, KSA, bensenouci@ieee.org
SIEEE Originally from: Electrical Engineering Department,
Ecole Nationale Polytechnique, Algiers, Algeria

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Abstract. A high performance motor drive requires fast step-tracking response with acceptable overshoot, minimum speed-dip and restore-time following a step load change, and zero steady-state error in the command tracking and load regulation. In this paper, a comparative study is carried out between two output-feedback control techniques to achieve a high performance induction motor. The first, named Discrete-Time Dynamic Programming (DTDP) output feedback, uses historical data from the controlled inputs and outputs of the motor and an optimization technique, dynamic programming algorithm, to obtain an optimum design of the needed constant output feedback gain matrix. In the last, named Linear Matrix Inequalities (LMI) output-feedback, the reduction of the disturbance on the motor speed is done through the minimization of the H-infinity (H∞) norm using Linear Matrix Inequality. The design procedure is based on the linearization of the motor nonlinear current-model around a selected operating point. The system performance of the motor equipped with DTDP and LMI controllers is analyzed using diverse tests namely, load disturbance (regulation and tracking) and parameters variation. For completeness, the performance of a conventional Proportional-Integral (PI) controller are also included for comparison purposes. The results are very encouraging to pursue further this study.
***Keywords*:** Output Feedback, Linear Matrix Inequalities (LMI), H-infinity (H∞) Control, Speed Control, and Induction Motor.

List of Symbols

n_p	**Number of Poles**	**Ψr**	**rotor flux:** $\psi_r = \sqrt{x_1^2 + x_2^2}$
R_r	**Rotor Resistance**	**Tr**	**rotor time-constant**
L_r	**Rotor Inductance**	**ψdr**	**d-axis rotor flux**
R_s	**Stator Resistance**	**ψqr**	**q-axis rotor flux**
L_s	**Stator Inductance**	**Ids**	**d-axis stator current**
L_{sr}	**Mutual Inductance**	**Iqs**	**q-axis stator current**
J	**Inertia**	**ωr**	**rotor speed**
D	**Viscous Coefficient**	**Tm**	**load (mechanical) torque**

1. Introduction

Induction motors (IM) [1] represent the workhorse of the industrial drive systems. They are less costly, more rugged, and more reliable than DC motors. The problems related to the induction motor are:

1. Stator and rotor parameters variation during motor operation
2. Difficulty in measuring the rotor time constant because of the temperature effect
3. Saturation effect on the rotor inductance and on the decoupling process between the rotor flux and torque
4. Nonlinear behavior and time-varying dynamics

Because of these problems, classical control design could not be done properly especially when parameter variation and load disturbance occur. To reduce the nonlinear coupling and fasten the transient response, usually field-oriented technique is used where a decoupling process between the torque and rotor flux is done.

In general, a high performance motor drive system is characterized by [2]:

- Fast step tracking response without overshoot
- Minimum speed dip and restore time, due to a step load change
- Achievement of zero steady-state error in the command tracking and load regulation

However, if regulation characteristics with small speed dip and short restore time following a step load change is required, relatively large overshoot, and short settling time in the speed tracking may result. So, to improve the system performance, the controller must be robust against speed variation and external perturbation.

Conventional Proportional-Integral-Derivative (PID) controller has been widely used in industrial applications due to its simple control algorithm and easy implementation. However, It is difficult and complex to design a high performance PID-controller [3] for induction motor drive systems because of system parameters variation and load disturbance change.

Modern control strategies involving intelligent techniques such as fuzzy logic control [4-5] and neural networks [6], represent attractive approaches. Besides, variable-structure control [7-8] is a robust technique but has a main drawback, the chattering. The later appears in the control input and makes such controller not attractive unless remedies are applied but at the expense of lowering the controller robustness.

State feedback [9] control requires all states to be measurable that is usually not the case unless observers are used that add to the complexity of the overall system. The output-feedback [10-12], however, requires only measurable system outputs to be used and thus made attractive in industrial control engineering area.

Linear Matrix Inequalities (LMIs) [13-14] have emerged as powerful design tools in areas such as control engineering. Three factors make LMI techniques appealing:

1. A variety of design specifications and constraints can be expressed as LMIs.
2. Once formulated in terms of LMIs, a problem can be solved *exactly* by efficient convex optimization algorithms
3. While most problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical "analytical" methods.

Many control problems and design specifications have LMI formulations. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control, H_∞-control, covariance control, etc. Further applications of LMIs arise in estimation, identification, optimal design, structural design, matrix scaling problems, and so on. The main strength of LMI formulations is the ability to combine various design constraints or objectives in a numerically tractable manner.

In this paper, two output-feedback design strategies are presented. In the first, Discrete-Time output-feedback [10] optimized via Dynamic Programming ((DTDP) and uses historical data from the system control inputs and outputs. In the second, Linear Matrix Inequalities output-feedback (LMI) where a minimization of the $H\infty$-norm is done using linear matrix inequalities technique. The design strategies are based on a linearized model around a selected operating point and severe tests, namely load disturbance (regulation and tracking) and parameters variation, were applied to both controlled systems and to the conventional Proportional-Integral (PI) controller for comparison purposes. MATLAB routines and LMI toolbox [15] were extensively used.

2. System Modeling

Fig. (1) shows the block diagram of a current controlled induction motor circuit [5]. The state space model is given by

$$\frac{d\mathbf{x}}{dt} = \mathbf{f(x)} + \mathbf{B_1}\,\mathrm{R} + \mathbf{Bu} \tag{1}$$

Where

$$\mathbf{x}=[\psi_{dr}\ \psi_{qr}\ \omega_r]^T$$
$$\mathrm{R}=T_m$$
$$\mathbf{u}=[I_{ds},\ I_{qs}]^T$$

And

$$\begin{aligned}
\frac{dx_1}{dt} &= -\frac{1}{T_r}x_1 + n_p(\omega_{syn} - x_3)x_2 + \frac{L_{sr}}{T_r}I_{ds} \\
\frac{dx_2}{dt} &= -\frac{1}{T_r}x_2 + n_p(-\omega_{syn} + x_3)x_1 + \frac{L_{sr}}{T_r}I_{qs} \\
\frac{dx_3}{dt} &= \frac{n_p L_{sr}}{J L_r}(x_1 I_{qs} - x_2 I_{ds}) - \frac{D}{J}x_3 - \frac{T_m}{J}
\end{aligned} \tag{2}$$

Rotor flux: $\psi_r = \sqrt{x_1^2 + x_2^2}$

Motor speed: $\omega_r = x_3$

The rotor time-constant $T_r = L_r/R_r$

The speed of the synchronously reference frame is taken as $\omega_{syn}=2\pi 50$ rad/sec. The system data are given in Table (1).

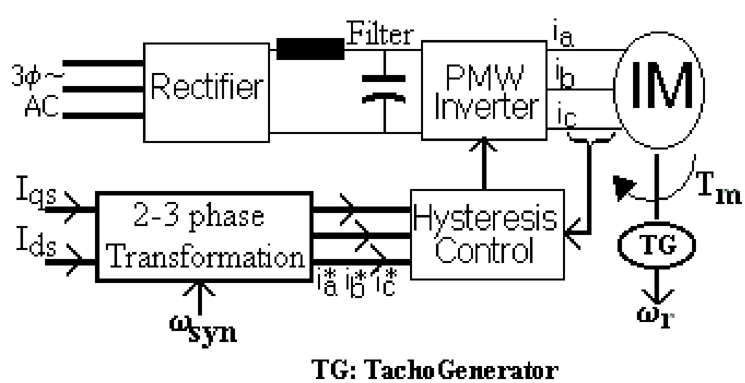


Fig. (1). Induction motor block diagram.

Table (1). Model Parameter Values.

Parameters	Values
Number of Poles, n_p	2 poles
Rotor Resistance, R_r	3.805 Ω
Rotor Inductance, L_r	0.274 H
Stator Resistance, R_s	4.85 Ω
Stator Inductance, L_s	0.274 H
Mutual Inductance, L_{sr}	0.258 H
Inertia, J	0.031 Kg.m^2
Viscous Coefficient, D	0.008 N/sec

3. Discrete-Time Dynamic-Programming (DTDP) Output-Feedback

The state-space model given in (1)-(2) is first linearized around an operating point then a discrete-time model [1] is derived as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{\Phi}\,\mathbf{x}_k + \mathbf{\Delta}\,\mathbf{u}_k \\ \mathbf{y}_{k+1} = \mathbf{C}\,\mathbf{x}_k + \mathbf{D}\mathbf{u}_k \end{cases} \tag{3}$$

where,

$\mathbf{x}_k=\mathbf{x}(kT_s)$, the state variable specified at kT_s, k=0,1, … etc.

$\mathbf{u}_k=\mathbf{u}(kT_s)$, the control input specified at kT_s, k=0,1, … etc.

$\mathbf{y}_k=\mathbf{y}(kT_s)$, the control input specified at kT_s, k=0,1, … etc.

$\mathbf{\Phi}$, $\mathbf{\Delta}$ are the state transition and input driving matrices, respectively.

The DTDP block diagram is as shown in Fig. (2) with $\mathbf{K}(s)=\mathbf{F_0}$, a constant matrix**,** y the measured output, and u is the control input.

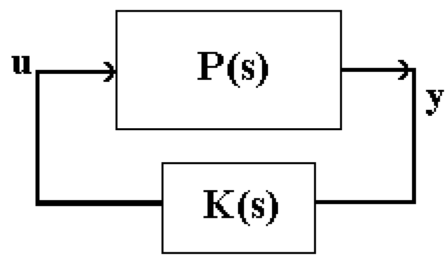


Fig. (2). Block diagram for DTDP.

The state prediction equation of the discrete-time linear model described in [10,12] can take the form:

$$\mathbf{x}_{k+1} = \mathbf{F}_5 w_k + \mathbf{F}_4 \mathbf{u}_k \tag{4}$$

The output-prediction equation has the form:

$$\mathbf{y}_{k+1} = \boldsymbol{\alpha}\,\mathbf{z}_k + \boldsymbol{\beta}\,\mathbf{v}_k \tag{5}$$

The prediction equation of the augmented vector $\mathbf{w_k}$ is

$$\mathbf{w}_{k+1} = \boldsymbol{\theta}\mathbf{w}_k + \boldsymbol{\Omega}\mathbf{u}_k \tag{6}$$

Where

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{y}_k & \mathbf{y}_{k-1} & \cdots & \mathbf{y}_{k-N+1} \end{bmatrix}^T$$

$$\mathbf{v}_k = \begin{bmatrix} \mathbf{u}_{k-1} & \mathbf{u}_{k-2} & \cdots & \mathbf{u}_{k-N+1} \end{bmatrix}^T$$

$$\mathbf{w}_k = \begin{bmatrix} \mathbf{z}_k & \mathbf{v}_k \end{bmatrix}^T$$

With $()^T$: matrix/vector transpose.

The matrices α, β, θ, Ω are defined in [10,12]. N is the measurement number of the outputs and the inputs from $t=kT_s$ back to $t=(k-N+1)T_s$. The minimum number of previous measurement vectors N is selected such that N≥n/p where "p" is the number of outputs and "n" the dimension of Φ.

Equation (5) completely defines the process dynamics without reference to the state vector x.

A state feedback optimal control law $\mathbf{u}_k = \mathbf{F}_s \mathbf{w}_k$ is determined from the minimization of the quadratic-performance index of the form:

$$\mathbf{J}_S = \sum_{k=0}^{r} \left[\mathbf{x}_{k+1}^T \mathbf{Q}_s \mathbf{x}_{k+1} + \mathbf{u}_k^T \mathbf{H}_s \mathbf{u}_k \right] \tag{7}$$

Where r represents the last stage in Dynamic Programming.
Similarly, an output feedback optimal control law

$$\mathbf{u}_k = \mathbf{F}_o \mathbf{w}_k \tag{8}$$

is determined from the minimization of the quadratic-performance index of the form:

$$\mathbf{J} = \sum_{k=0}^{r} \left[\mathbf{w}_{k+1}^T \mathbf{Q}_o \mathbf{w}_{k+1} + \mathbf{u}_k^T \mathbf{H}_o \mathbf{u}_k \right] \tag{9}$$

The two performance indexes given by (7) and (8) are equivalent if (4) is substituted into (7) to get

$$\mathbf{J}_o = \sum_{k=0}^{r} \left[\mathbf{w}_k^T \mathbf{Q} \mathbf{w}_k + \mathbf{u}_k^T \mathbf{R} \mathbf{w}_k + \mathbf{u}_k^T \mathbf{S} \mathbf{u}_k \right] \tag{10}$$

Where

$$\begin{cases} \mathbf{Q} = \mathbf{F}_5^T \mathbf{Q}_s \mathbf{F}_5 \\ \mathbf{R} = 2\mathbf{F}_4^T \mathbf{Q}_s \mathbf{F}_5 \\ \mathbf{S} = \mathbf{F}_4^T \mathbf{Q}_s \mathbf{F}_4 + \mathbf{H}_s \end{cases} \tag{11}$$

To reach the global optimum of J_0 given by (10), the weight matrices H_s and Q_s are assumed to be symmetric positive definite matrices [10]. For stability analysis, the closed loop eigenvalues of DTDP can be determined from (6). On the basis of assumed sampling time interval T_s, the optimization problem is thus defined as:

$$\text{Find } F_0 \text{ that minimizes } \mathbf{J}_o = \sum_{k=0}^{r} \left[\mathbf{w}_k^T \mathbf{G} \mathbf{w}_k \right]$$

$$\text{with respect to } \mathbf{u}_k = \mathbf{F}_o \mathbf{w}_k$$

$$\text{where } \mathbf{G} = \mathbf{Q} + \mathbf{F}_o^T \mathbf{R} + \mathbf{F}_o^T \mathbf{R} \mathbf{F}_o .$$

To evaluate the output feedback control gain matrix F_o, Dynamic Programming (DP) technique is applied here to minimize J_o for several stages starting from initial stage k=0 and moving backward until stage k=r. If r is large enough, the DP algorithm converges to a constant feedback matrix. The multi-stage dynamic programming algorithm [10] is summarized as:

<u>Step1</u>: Initialization process

$\sigma = 0$

Compute

$$\eta = R + 2\Omega^T \sigma\theta$$
$$\mu = S + \Omega^T \sigma\Omega$$
$$F = -0.5*\mu^{-1}\eta$$
$$k=1$$

<u>Step 2</u>: Iterate while $k>0$ & $|F-F_0|$ > tolerance, do

$$\{ \quad F_0 = F$$
$$\sigma = Q + \theta^T \sigma\theta + F^T \eta + F^T \mu F$$
$$\mu = S + \Omega^T \sigma\Omega$$
$$\eta = R + 2\Omega^T \sigma\theta$$
$$F = -0.5*\mu^{-1}\eta$$
$$k = k+1 \qquad \}$$

4. Linear Matrix Inequality (LMI) Robust Output Feedback Control

Fig. (3) shows the standard representation of the output-feedback control block diagram for the LMI-based robust control where **P(s)** represents the plant while **K(s)** represents the controller.

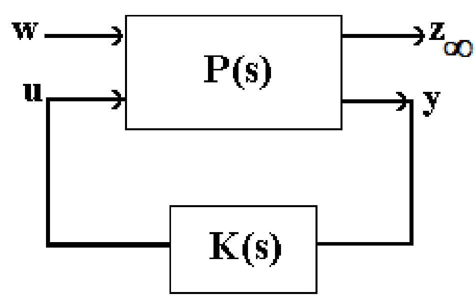


Fig. (3). Block diagram for LMI Robust output feedback control Let.

$$\mathbf{P(s)}: \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ \mathbf{z}_\infty = \mathbf{C}_z\mathbf{x} + \mathbf{D}_{z1}\mathbf{w} + \mathbf{D}_{z2}\mathbf{u} \\ \mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y\mathbf{w} \end{cases} \tag{12}$$

and

$$\mathbf{K(s)}: \begin{cases} \dot{\zeta} = \mathbf{A}_K\zeta + \mathbf{B}_K\mathbf{y} \\ \mathbf{u} = \mathbf{C}_K\zeta + \mathbf{D}_K\mathbf{y} \end{cases} \tag{13}$$

be state-space realizations of the plant **P(s)** and controller **K(s)**, respectively, and let

$$\begin{cases} \dot{\mathbf{x}}_{CL} = \mathbf{A}_{CL}\mathbf{x}_{CL} + \mathbf{B}_{CL}\mathbf{w} \\ \mathbf{z}_\infty = \mathbf{C}_{CL}\mathbf{x}_{CL} + \mathbf{D}_{CL}\mathbf{w} \end{cases} \tag{14}$$

be the corresponding closed-loop state-space equations with

$$\mathbf{x}_{CL} = [\mathbf{x} \quad \zeta]^T .$$

The design objective for finding K(s) is to optimize the H_∞-norm of the closed-loop transfer G(s) from w to z_∞, i.e.,

$$\mathbf{G}(s)=\mathbf{C}_{CL}\,(s\mathbf{I}-\mathbf{A}_{CL})^{-1}\mathbf{B}_{CL}+\mathbf{D}_{CL} \tag{15}$$

using LMI technique [14]. This can be fulfilled if and only if there exists a symmetric matrix X such that the following linear matrix inequalities are satisfied

$$\begin{pmatrix} \mathbf{A}_{CL}\mathbf{X}+\mathbf{X}\mathbf{A}_{CL}^{T} & \mathbf{B}_{CL} & \mathbf{X}\mathbf{C}_{CL}^{T} \\ \mathbf{B}_{CL}^{T} & -\mathbf{I} & \mathbf{D}_{CL}^{T} \\ \mathbf{C}_{CL}\mathbf{X} & \mathbf{D}_{CL} & -\gamma^{2}\mathbf{I} \end{pmatrix} < 0 \tag{16}$$

$$\mathbf{X} > 0$$

The H_∞-norm of a stable transfer function G(s) is its largest input/output RMS gain over all u with the random Mean Square (RMS) different from zero, i.e., RMS≠0,

$$\|\mathbf{G}\|_\infty = \sup_{\substack{\mathbf{u}\in L \\ \mathbf{u}\neq 0}} \frac{\|\mathbf{z}_\infty\|_L}{\|\mathbf{w}\|_L} \tag{17}$$

where *L* is the space of signals with finite energy and z is the output of the system G for a given disturbance w. It is one of disturbance rejection, i.e., minimization of the effect of the worst-case disturbance on the output. Equations (16) are being solved using the Matlab routine *hinflmi*.

5. Simulation and Test Results

The model of the induction motor is a linear one obtained using a sampling time selected using trial-and-error technique. It is selected neither too small to induce a large amount of computations nor too large to end up in a numerical instability. The value, T_s =0.01 second, was found adequate. The following tests are carried out for the three cases namely, the machine is driven by a conventional Proportional-Integral (PI), a Discrete-Time Dynamic Programming (DTDP) and finally a Linear Matrix Inequality (LMI) based controllers:

- Step changes in load torque
- Tracking behavior in load torque
- Change in system parameters

The continuous open-loop linear state-space system of the machine (or plant P (s)) are:

$$\mathbf{A_p} = \begin{bmatrix} -\frac{1}{T_r} & n_p(\omega_s - x_0(3)) & -n_p x_0(2)) \\ -n_p(\omega_s - x_0(3)) & -\frac{1}{T_r} & n_p x_0(1)) \\ \frac{n_p L_{sr} Iqs0}{JL_r} & -\frac{n_p L_{sr} Ids0}{JL_r} & -\frac{D}{J} \end{bmatrix}$$

$$\mathbf{B_p} = \begin{bmatrix} 0 & \frac{L_{sr}}{T_r} & 0 \\ 0 & 0 & \frac{L_{sr}}{T_r} \\ -\frac{1}{J} & -\frac{n_p L_{sr} x_0(2)}{JL_r} & \frac{n_p L_{sr} x_0(1)}{JL_r} \end{bmatrix}$$

$$\mathbf{C_p} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{x_0(1)}{\sqrt{x_0^2(1) + x_0^2(2)}} & \frac{x_0(2)}{\sqrt{x_0^2(1) + x_0^2(2)}} & 0 \end{bmatrix} \qquad \mathbf{D_p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Where

$$\mathbf{x} = \begin{bmatrix} \psi_{dr} & \psi_{qr} & \omega_r \end{bmatrix}^T \quad \mathbf{u} = \begin{bmatrix} T_m & I_{ds} & I_{qs} \end{bmatrix}^T \quad \mathbf{y} = \begin{bmatrix} \omega_r & \psi_r \end{bmatrix}^T$$

Or,

$$\mathbf{A_p} = \begin{bmatrix} -13.89 & 157 & 0.06 \\ -157 & -13.89 & 5.70 \\ 0 & -671.3 & -0.26 \end{bmatrix} \qquad \mathbf{B_p} = \begin{bmatrix} 0 & 3.58 & 0 \\ 0 & 0 & 3.58 \\ -32.26 & 1.83 & 173 \end{bmatrix}$$

$$\mathbf{C_p} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -0.01 & 0 \end{bmatrix} \qquad \mathbf{D_p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{x_0} = \begin{bmatrix} 2.85 & -0.030 & 78.5 \end{bmatrix}^T$$

Case 1: Conventional Proportional-Integral (PI) Controller

The system driven by a PI controller is shown in Fig. (4) where

$$\mathbf{w} = T_m \qquad \mathbf{u} = \begin{bmatrix} I_{ds} & I_{qs} \end{bmatrix}^T \qquad \mathbf{y} = \begin{bmatrix} \omega_r & \psi_r \end{bmatrix}^T$$

The controller gains used are:

$$\mathbf{K}_p = \begin{bmatrix} K_{p1} \\ K_{p2} \end{bmatrix} \qquad \mathbf{K}_i = \begin{bmatrix} K_{i1} \\ K_{i2} \end{bmatrix}$$

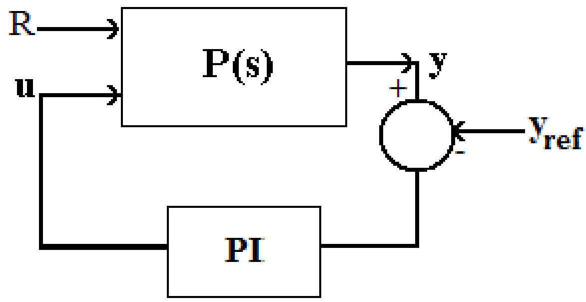


Fig. (4). System driven by a Proportional-Integral (PI) Controller.

Case 2: Discrete-Time Dynamic Programming (DTDP) output feedback

An DTDP is designed with:

N=5 number of historical data
r=51 number of stages before DP convergence
p=2 dimension of $u=[I_{ds}\ I_{qs}]^T$
m=2 dimension of $y=[\psi_r\ \omega_r]^T$.
and
$z=\omega_r$
$w=T_m$.

The discrete closed-loop eigenvalues are:

λ□□□$_{F0}$ = { -0.09±j0.87, -0.7, 0.6, -0.36±j0.24, 0.06±j0.4, 0.34±j0.15, $6*10^{-5}$, $9*10^{-9}\pm j6*10^{-5}$, $1.8*10^{-5}$,$-1.3*10^{-9}\pm j1.8*10^{-5}$, $-1.8*10^{-5}$, $-6*10^{-5}$ }

With $j^2=-1$

The magnitude of the discrete dominant one is: 0.876

$$\mathbf{F_{01}} = [\,0.0002\ -0.0015\ -0.0134\quad 0.0016\quad -0.0098\quad 0.0005\ \ldots$$
$$0.0005\ -0.0013\ -5.9*10^{-5}\ \ -5.9*10^{-5}\ -0.0035\ \ -0.0362\ \ldots$$
$$-0.0005\ -0.0164\ -5.97*10^{-6}\ \ 0.0008\quad -6*10^{-6}\ \ -0.0001]$$
$$\mathbf{F_{02}} = [-0.0606\ \ -0.0026\ -0.0412\ -0.0070\ -0.0707\ \ 0.0024\ \ \ldots$$
$$-0.0616\quad 0.0040\quad 0.0028\ -0.0078\ -0.0365\ \ -0.2442\ \ldots$$
$$-0.0215\ -0.2048\ \ -0.0036\ \ \ -0.1035\ \ 1.9*10^{-5}\ \ 0.0048]$$

$$\mathbf{F_0} = \begin{bmatrix} \mathbf{F_{01}} \\ \mathbf{F_{02}} \end{bmatrix}$$ with dimension: 2x18

Case 3: LMI Output Feedback (LMI)

A controller **K**(s) is designed by reducing the H$_\infty$-norm below some specified value γ. The selected value was 10 but it was reduced to □= 5.3.

The obtained controller **K**(s) matrices are:

$$\mathbf{A_K} = \begin{bmatrix} -153 & 458 & 317 \\ -52.6 & -518 & 8.6 \\ 97.5 & 510 & -228 \end{bmatrix} \qquad \mathbf{B_K} = \begin{bmatrix} -420 & 250 \\ 526 & -8.15 \\ -540 & -261 \end{bmatrix}$$

$$\mathbf{C_K} = \begin{bmatrix} 0.055 & 0.239 & 0.099 \\ 2.573 & -0.134 & 2.99 \end{bmatrix} \qquad \mathbf{D_K} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Its eigenvalues are: $\boldsymbol{\lambda}_{\mathbf{K}} = [-542\quad -114\quad -242]^T$

The closed-Loop system matrices are:

$$\mathbf{A_{cl}} = \begin{bmatrix} -13.9 & 157 & 0.06 & 0.19 & 0.85 & 0.35 \\ -157 & -13.9 & 5.7 & 9.22 & 0.48 & 10.7 \\ 0 & -671 & -0.26 & 445 & -22.7 & 518 \\ 250 & -2.6 & -420 & -152 & 458 & 317 \\ -8.14 & 0.086 & 526 & -52.6 & -518 & 8.59 \\ -261 & 2.77 & -540 & 97.5 & 510 & -228 \end{bmatrix} \qquad \mathbf{B_{cl}} = \begin{bmatrix} 0 \\ 0 \\ -32 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

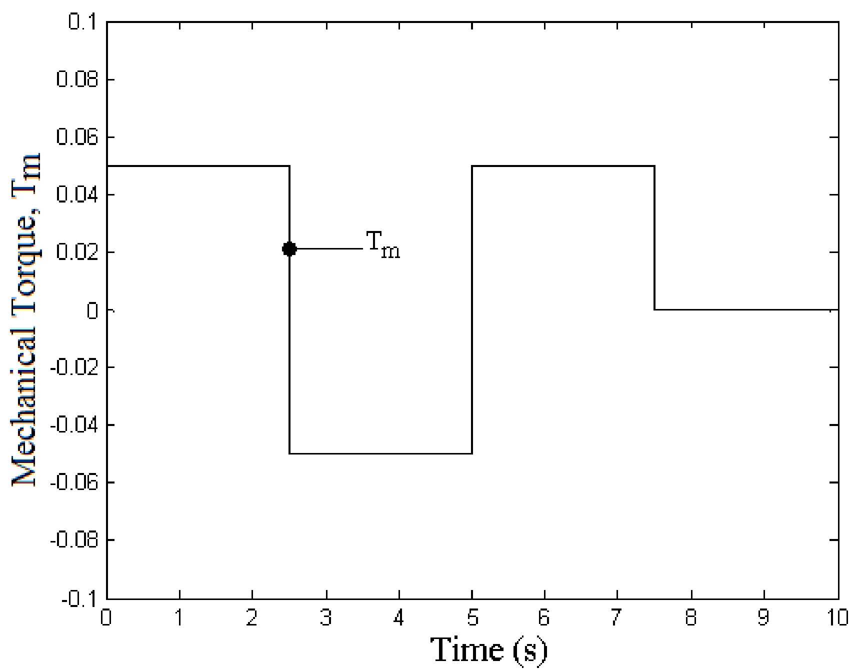
$$\mathbf{C_{cl}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{D_{cl}} = \begin{bmatrix} 0 \end{bmatrix}$$

The closed-loop eigenvalues are:

$$\boldsymbol{\lambda}_{\mathbf{cl}} = \begin{bmatrix} -256 - j666 & -256 + j666 & -3.94 & -7.2 - j157 & -7.2 + j157 & -5 \end{bmatrix}^T$$

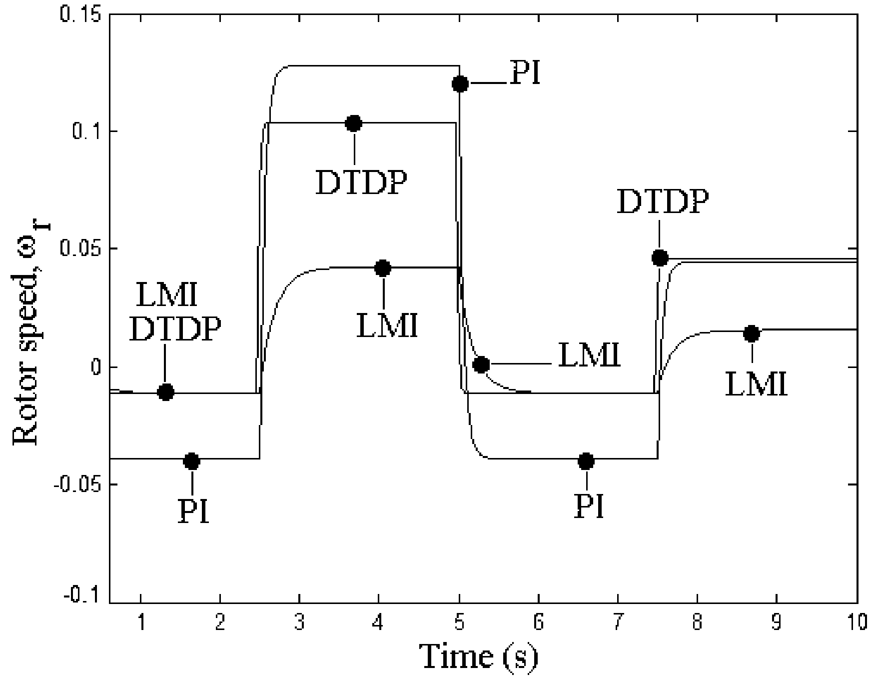
Test 1: Load torque step changes:

The load torque T_m is varied in a step-wise fashion as seen in Fig. (5 a). The time responses of the motor speed ω_r for CPI, DTDP and LMI, are depicted in Fig. (5 b).



(a) Load torque variation

Fig. (5). Responses to step changes in T_m



(b) Rotor speed
Fig. (5). Responses to step changes in T_m.

Test 2: Tracking behavior:

The motor is being disturbed from its steady-state with a variation in T_m (tracking) as depicted in Fig. (6). The time responses of the motor speed, ω_r, for CPI, DTDP and LMI are depicted in Fig. (7).

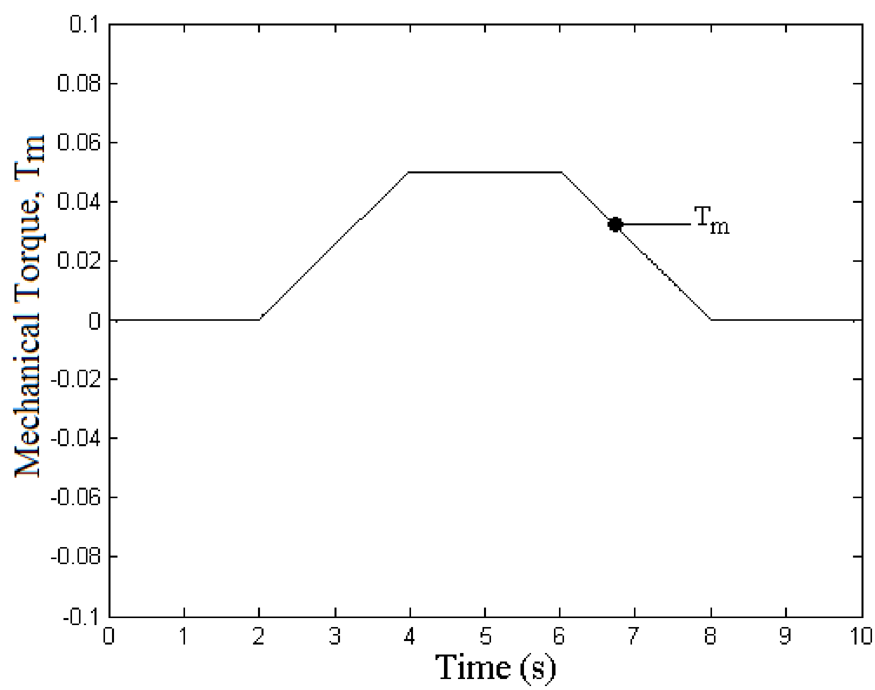


Fig. (6). Load torque tracking behavior.

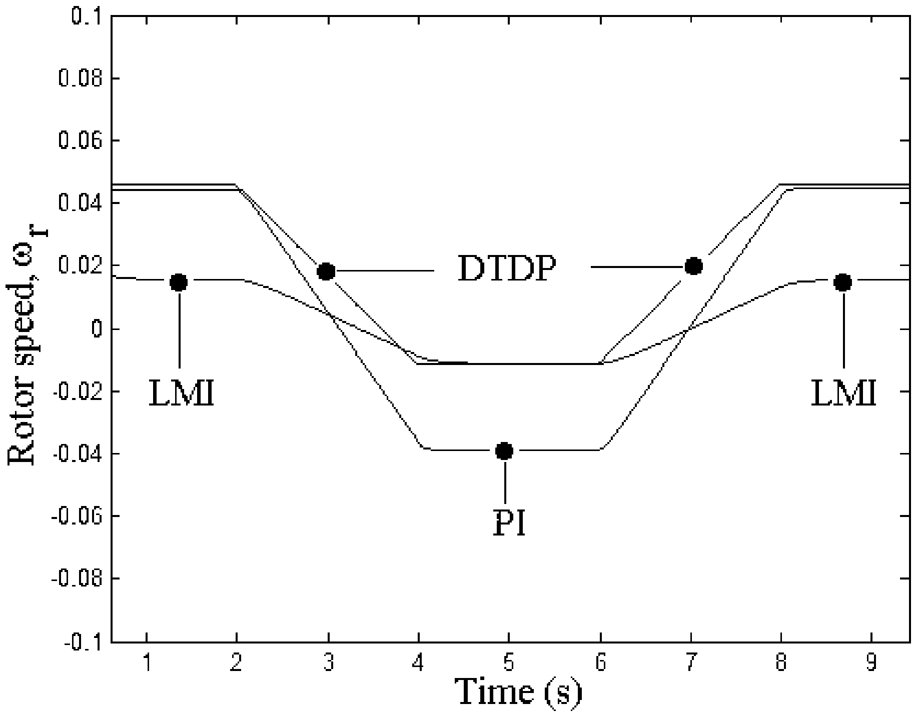


Fig. (7). Responses to tracking behavior.

Test 3: Parameters Variation:

Three motor parameters were increased by 50% from their nominal values. They are: the rotor time constant T_r, the damping coefficient D, and the inertia constant J. This large parameter change that might not be realistic is used to demonstrate how far the proposed design is valid and acceptable. It is motivated by the practical difficulty encountered in determining the exact values of the rotor parameters especially in a squirrel cage induction motor with deep-bar double-cage rotor designs.

In this test, the load torque T_m is increased by 5% and the time responses of the motor speed, ω_r, for CPI, DTDP and LMI are depicted in Fig. (8).

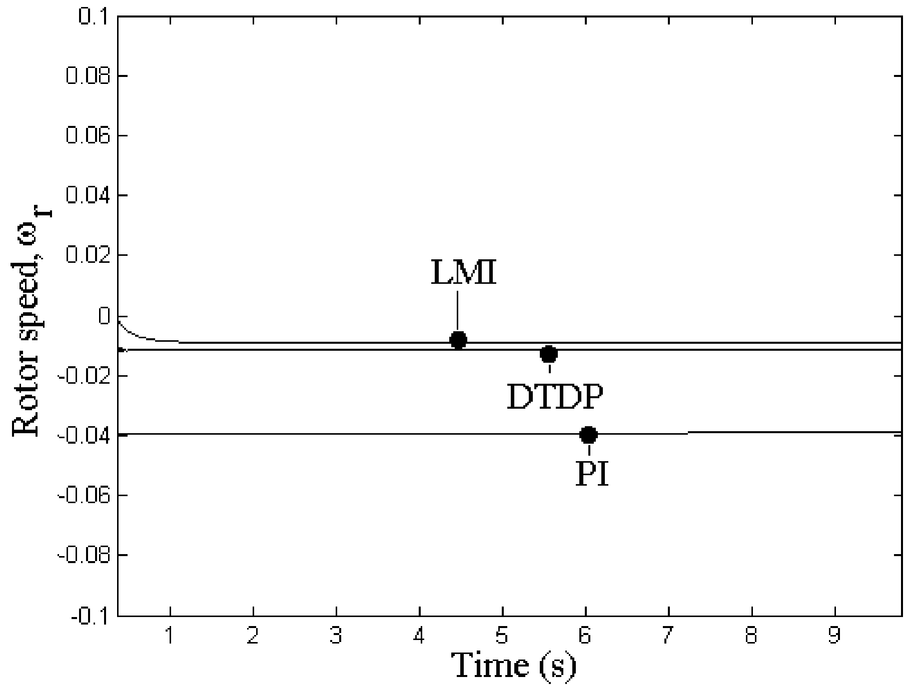


Fig. (8). Responses to Parameters Variation.

Remarks on the Results

From the simulation results, it is clear that the system equipped with each of the three controllers shows good response. However, LMI shows superiority over DTDP and PI from deep/rise of the motor speed ω_r following the disturbance in the load torque Tm, point of view that is it shows the lowest amount in dips/rises in ω_r. Besides, the controlled system shows fast response without oscillations or overshoots. The LMI controller can be made faster by proper selection of its parameters.

6. Conclusion

This paper has presented the design steps for two output-feedback controllers. The first uses the Discrete-Time Dynamic Programming (DTDP) whereas, the second uses the Linear Matrix Inequalities (LMI) techniques. The Conventional Proportional-Integral (PI) case results are also presented for comparison purposes. The two controllers are used to improve the transient response and to minimize the induction motor speed dips and rises following load torque disturbances and system parameters variation. The tests have shown improved performance for both controllers. It was seen that LMI is much robust as compared to DTDP and PI.

As an extension to this work, the LMI can be investigated deeply to improve the system response more by a better selection of γ and/or the use of pole placement technique. The test on the nonlinear model and other nonlinearities such as limitation in the control input will be further investigated. Finally, on-line identification will be also looked into.

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تصميم متحكم متقطع – زمني مبني على البرمجة – الديناميكية وآخر متين مبني على تباين المصفوفات الخطية لحلقة تغذية خلفية للتحكم في سرعة محرك حثي

أحمد الخير بن سنوسي

عضو سامي بجمعية المهندسين الكهربائيين والإلكترونيين العالمية (SIEEE)

قسم الهندسة الكهربائية، كلية الهندسة، جامعة القصيم

القصيم، المملكة العربية السعودية

bensenouci@ieee.org

(قدم للنشر في ٢٠٠٩/٣/٢٦؛ وقبل للنشر في ٢٠٠٩/٦/١م)

ملخص البحث. يتطلب محرك الجر ذو كفاءة عالية سرعة الاستجابة لتتبع خطوة التغيير والمتابعة بتجاوز مقبول وأدنى قيمة لسرعة الانخفاض وزمن – الإعادة جراء خضوع المحرك لخطوة في الحمل مع انعدام الخطاء أثناء متابعة القيمة المطلوبة والتغيير في الحمل.في هذا البحث، نقدم دراسة مقارنة بين طريقتين للتحكم في حلقة تغذية خلفية للحصول على محرك حثي ذو كفاءة عالية.

الأول ويسمى متحكم مقطع–زمني ذو حلقة تغذية خلفية (DTDP) ويستخدم بيانات سابقة من دخل وخرج المحرك والطريقة المستخدمة لتحقيق الأمثلية والمعروفة بالبرمجة–الديناميكية للحصول على أفضل تصميم لمصفوفة الكسب المرغوبة وهي ثابتة القيمة.

أما الآخر والمسمى متحكم ذو حلقة تغذية خلفية مصمم باستخدام تباين المصفوفات الخطية (LMI)، حيث أن دراسة تقليص أثر الاضطراب (عزم الحمل) على سرعة دوران المحرك تتم بالحصول على أدنى قيمة للمعيار$H\infty$ (إتش – اللاتناهي) باستخدام تباين المصفوفات الخطية.

طريقة التصميم مبنية على إيجاد النموذج الخطي للمحرك من نموذجه اللاخطي للتيار وذلك حول نقطة تشغيل مختارة. دراسة أداء المحرك مزود بالمتحكم DTDP وLMI تتم عن طريق إخضاع المحرك الى عدة اختبارات منها اضطراب في الحمل (التنظيم والتتبع) وتغيير في البارامترات. وإتماما لهذه الدراسة، تم إضافة دراسة أداء المتحكم التقليدي التناسبي–التكاملي (PI) بغرض المقارنة. النتائج مشجعة جدا لمتابعة هذه الدراسة.