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A Comparative Study Between *Hi/H2/MOC* LMI-based Iterative PID Controllers for Speed and Voltage Control of a Sample Power System

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Abstract. This paper presents a comparative study between three Linear Matrix Inequality (LMI)-based iterative multivariable Proportional-Integral-Derivative (PID) controllers; PID design using H_∞-norm, named *Hi*, PID design using H_2-norm, named *H2*, of the system transfer function, PID design with Maximum Output Control (MOC), named *Max*, and the classical LMI-based robust output feedback controller using H_∞-norm, named *ROB*. Multivariable PID is considered here because of its wide use in the industry, simple structure and easy implementation. It is also preferred in plants of higher order that cannot be reduced and thus require a controller of higher order such as is the case for the classical robust H_∞ output feedback controller whose order is the same as that of the plant. LMI technique is selected because it allows easy inclusion of divers system constraint requirements that should be fulfilled by the controller, and thus make its design very efficient. The duty of each of the controllers is to drive a single-generator connected to a large power system via a transformer and a transmission line. The generator is equipped with its speed/power (governor) and voltage (exciter) control-loops that are lumped in one block. The errors in the terminal voltage and in the output active power, with respect to their respective references, represent the controller inputs and the generator-exciter voltage and governor-valve position represent the controller outputs. A comparative study is carried out using the named controllers (*Hi*, *H2*, *Max*, *ROB*). Divers tests are applied, namely, step-change and tracking in the references of the controlled variables, and variation in some plant parameters, to demonstrate the controllers effectiveness. Encouraging results are obtained that motivate for further investigations.

***Keywords*:** Linear matrix inequality, power system, robust output feedback control, H_∞-control with PID, H_2-control with PID, Maximum output with PID.

List of Symbols

vd, q	stator voltage in d-axis and q-axis circuit
V_t	terminal voltage
ψ_{fd}	field flux linkage
x_{ad}	stator-rotor mutual reactance
x_{fd}	self reactance of filed winding
V_{fd}	field voltage
r_{fd}	field resistance
e	busbar voltage resistance
U_e	exciter input
δ	rotor angle
T_e/T_m	electrical / mechanical torque
P_s	steam power
H	inertia constant
ω	angular frequency of rotor
ω_0	angular frequency of the infinite busbar
K_d	mechanical damping torque coefficient
T_d	damping torque coefficient due to damper windings
P_t	real power output at the generator terminals
τ_e	exciter time constant
τ_g	governor valve time constant
τ_b	turbine time constant
U_g	governor input
G_v	governor valve position
K_v	valve constant

1. Introduction

In a power system, the regulators of the synchronous machines determine power system voltage/frequency profile. Conventional regulators [1-3] such as IEEE types are characterized by low frequency oscillations and slow response. Other control signals are usually added to improve the performance but at the expense of a more complicated system.

Conventional Proportional-Integral-Derivative (PID) controller is widely used in the industry owing to its simple structure, easy implementation, and found to be adequate for most plants. However, it is not robust to disturbances in the controlled variables and system parameters change [4-6]. Variable Structure Control (VSC) technique represents a robust control technique but it has a main drawback; the chattering (higher switching). Because of the limitation of the physical actuators, it is impossible to achieve the necessary higher switching. Besides, the chattering appears in the control input, and makes such controller not attractive unless remedies are applied, but at the expense of lowering the controller robustness [7-10]. Optimal control theory [4,11-12] was also investigated and applied in industrial processes. State-feedback control is attractive but requires all states to be measurable that is usually not the case unless observers are used that add to the complexity of the overall system. This burden is reduced by using output feedback control instead. The later requires only measurable system outputs to be used and thus made more attractive in industrial control engineering area. Thus, efficient controllers are desirable to improve the power system performance through the control of the generator voltage and speed, and to overcome limitations in stability boundaries caused by the use of larger generator size and longer transmission lines. Modern control strategies involving intelligent techniques such as fuzzy logic control and neural networks, represent attractive approaches but have also limitations [9,13-14]. Recently, Linear Matrix Inequality (LMI) technique [18-20] has emerged as powerful design tools. Many control problems and design specifications have LMI formulations. This is especially true for Lyapunov-based analysis and design, but also for optimal LQG control (H_2-control), robust H_∞-control, etc. The main strength of LMI formulations is its ability to combine various design constraints and/or objectives in a numerically tractable manner. The LMI theory offers powerful tools to attack different objectives such as:

- H_∞ performance (for tracking, disturbance rejection, or robustness aspects).
- *H2 performance (for LQG aspects).*
- *Robust pole placement specifications to ensure fast and well-damped transient responses.*
- *Maxim*um Output Feedback (MOC) control.

In robust control, it is customary to formulate the design specifications as abstract disturbance rejection objectives. The performance of a control system is then measured in terms of the closed-loop RMS gain from disturbances to outputs. While some tracking and robustness are best captured by an H_∞ criterion, noise insensitivity is more naturally expressed in LQG terms (H_2-performance), and transient behaviors are more easily tuned in terms of the system closed-loop damping. Classical H_∞-based robust output-feedback controller is widely preferred when the minimization of the effect of the disturbance on selected outputs is sought. However, due to its complexity in implementation and its high order, it is not highly desirable [8,9].

This paper presents a comparative study between three iterative LMI-based iterative multivariable PID controllers: PID using H_∞-norm of the system transfer function, abbreviated *Hi*; robust PID using H_2-norm of the same transfer function, abbreviated *H2*; PID with Maximum Output Control (MOC), abbreviated *Max*, and finally the classical LMI-based robust H_∞ output feedback controller, abbreviated *ROB* [21-23]. The main task of each of the controllers is to drive a single-generator connected to a large power system via a transformer and a transmission line [11]. The generator is equipped with its speed/power (governor) and voltage (exciter) control-loops. To show the effectiveness of each controller and to carry a comparative study, divers tests were applied, namely, step-change and tracking in the references of the controlled variables, and variation in some plant parameters.

2. System Modeling

Figure. (1) shows the block diagram of the sample controlled power system that comprises a steam turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The output real power P_t and terminal voltage V_t at the generator terminals are measured and fed to the controller. The outputs of the controller (system control inputs) are fed into the generator-exciter and governor-valve.

In the simulation studies described here, the nonlinear equations of the synchronous generator are represented by a third–order nonlinear model based on park's equations. The steam turbine, governor valve and exciter are each represented by a first order- model. The model equations are as follows [11]. The data are shown in the Appendix.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (x_6 - K_1 x_3 \sin x_1 - K_2 \sin x_1 \cos x_1 - (K_d + T_d) x_2) \dfrac{\omega_0}{2H} \\ \dot{x}_3 = \dfrac{\omega_0 r_{fd}}{x_{ad}} x_4 + K_3 x_3 - K_2 \sin x_1 \cos x_1 \\ \dot{x}_4 = \dfrac{-x_4}{\tau_e} + \dfrac{1}{\tau_e} U_e \\ \dot{x}_5 = \dfrac{-x_5}{\tau_g} + \dfrac{K_v}{\tau_g} U_g \\ \dot{x}_6 = \dfrac{-x_6}{\tau_b} + \dfrac{x_5}{\tau_b} \end{cases} \tag{1}$$

The output y_1, y_2 may be expressed in terms of these state variables by

$$\begin{cases} y_1 = P_t = K_1 x_3 \sin x_1 + K_2 \sin x_1 \cos x_1 \\ y_2 = V_t = (v_d^2 + v_q^2)^{1/2} \end{cases} \tag{2}$$

where

$$\begin{cases} v_d = K_5 \sin x_1 \\ v_q = K_6 x_3 + K_7 \cos x_1 \end{cases} \tag{3}$$

A linear Multi-Input Multi-output (MIMO) model of the generator system is required to design a controller for such system. It is derived from the system nonlinear model by linearizing the nonlinear equations (1)-(3) around a specific operating point. The linear state-space model (4) is derived next where the variables shown represent small displacements around the selected operating point.

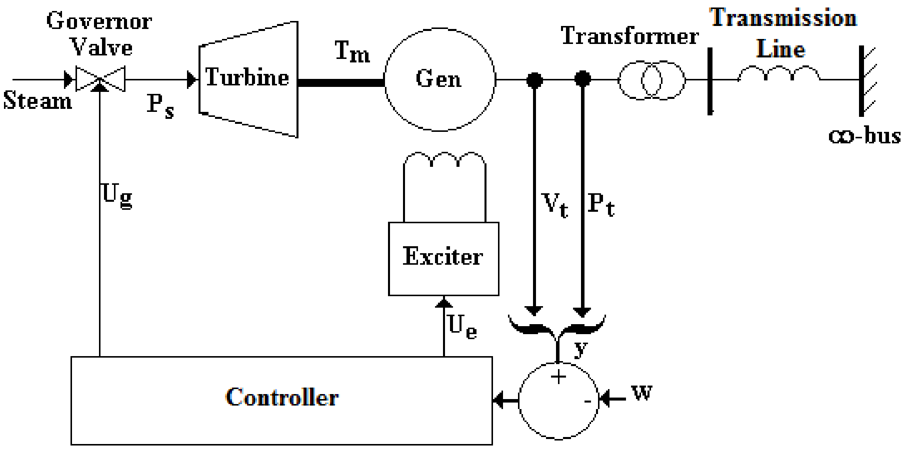

Fig. (1). Controlled sample power system.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{4}$$

The matrices A, B, C and D have the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ K_8 & \frac{-(K_d + T_d)\omega_0}{2H} & K_9 & 0 & 0 & \frac{\omega_0}{2H} \\ K_{10} & 0 & K_3 & \frac{\omega_0 r_{fd}}{x_{ad}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_b} & \frac{-1}{\tau_b} \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_e} & 0 \\ 0 & \frac{K_g}{\tau_g} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} K_{11} & 0 & K_{12} & 0 & 0 & 0 \\ K_{13} & 0 & K_{14} & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where

$x = \begin{bmatrix} \delta & \dot{\delta} & \psi_{fd} & E_{fd} & P_s & T_m \end{bmatrix}^T$:	state variables vector
$u = \begin{bmatrix} U_e & U_g \end{bmatrix}^T$:	control input vector
$y = \begin{bmatrix} P_t & V_t \end{bmatrix}^T$:	output measurement vector
$P_t = K_{11} x_1 + K_{12} x_3$:	output power
$V_t = K_{13} x_1 + K_{14} x_3$:	terminal voltage

3. Robust H∞ Output Feedback Controller

Figure. (2) shows a modified representation of the output-feedback control block diagram.

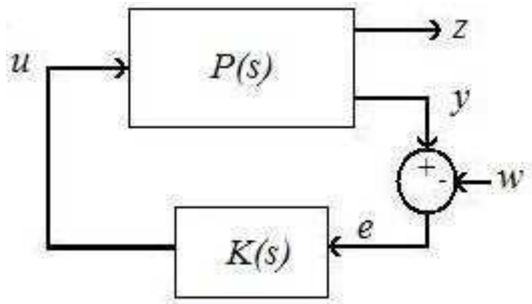

Fig. (2). Output feedback block diagram.

Where *P(s)* represents the plant whereas *K(s)* represents the controller to be designed. Let

Plant:
$$P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_z x + D_{z1} w + D_{z2} u \\ y = C_y x + D_y w \end{cases} \tag{5}$$

Controller:
$$K(s): \begin{cases} \dot{\zeta} = A_K \zeta + B_K e \\ u = C_K \zeta + D_K e \end{cases} \tag{6}$$

be the state-space realizations of the plant *P(s)* and the controller *K(s)*, respectively, and let

$$\begin{cases} \dot{x}_{CL} = A_{CL} x_{CL} + B_{CL} w \\ z = C_{CL} x_{CL} + D_{CL} w \end{cases} \tag{7}$$

be the corresponding closed-loop state-space equations with

$$\begin{cases} x_{CL} = [x \quad \zeta]^T \\ z = e = y - w \end{cases} \tag{8}$$

The design objectives for finding *K(s)* is to minimize the H_∞-norm of the closed-loop transfer function *G(s)* from *w* to *z*, i.e.,

$$G(s) = C_{CL} (s - A_{CL})^{-1} B_{CL} + D_{CL} \tag{9}$$

satisfies

$$\| G(s)_{zw} \|_\infty < \gamma$$

using LMI technique [12,15-17]. This can be fulfilled if and only if there exists a symmetric matrix X such that the following LMIs are satisfied

$$\begin{bmatrix} A_{CL}X + XA_{CL}^T & B_{CL} & XC_{CL}^T \\ B_{CL}^T & -I & D_{CL}^T \\ C_{CL}X & D_{CL} & -\gamma^2 I \end{bmatrix} < 0$$

$$X > 0 \tag{10}$$

4. PID Design with H∞

Consider the linear time-invariant state-space system given by

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{11}$$

With the following PID controller

$$u = F_1 y + F_2 \int_0^t y dt + F_3 \frac{dy}{dt} \tag{12}$$

Where

x	state variables
u	control inputs
y	outputs
A, B and C	matrices with appropriate dimensions
F_1, F_2, F_3	matrices to be designed.

Let

$$\begin{cases} z_1 = x \\ z_2 = \int_0^t y dt \end{cases}. \tag{13}$$

Denote $z = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T$. The variable z can be viewed as the state vector of a new system whose dynamics are governed by

$$\begin{cases} \dot{z}_1 = \dot{x} = Az_1 + Bu \\ \dot{z}_2 = y = Cz_1 \end{cases} \tag{14}$$

Or, in compact form,

$$\dot{z} = \bar{A}z + \bar{B}u \tag{15}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Combining (11) and (13) yields

$$\begin{cases} y = \begin{bmatrix} C & 0 \end{bmatrix} z \\ \int_0^t y dt = \begin{bmatrix} 0 & I \end{bmatrix} z \\ \dfrac{dy}{dt} = CAx + CBu = \begin{bmatrix} CA & 0 \end{bmatrix} z + CBu \end{cases} \quad (16)$$

Define

$$\bar{C}_1 = \begin{bmatrix} C & 0 \end{bmatrix}, \qquad \bar{C}_2 = \begin{bmatrix} 0 & I \end{bmatrix}, \qquad \bar{C}_3 = \begin{bmatrix} CA & 0 \end{bmatrix}$$

Then,

$$\bar{y}_i = \bar{C}_i z \qquad (i = 1-3)$$

If $(I - F_3CB)$ is invertible then from (12) and (16), one gets

$$u = \bar{F}\bar{y} \quad (17)$$

Where

$$\bar{y} = \begin{bmatrix} \bar{y}_1^T & \bar{y}_2^T & \bar{y}_3^T \end{bmatrix}^T \qquad \bar{C} = \begin{bmatrix} \bar{C}_1^T & \bar{C}_2^T & \bar{C}_3^T \end{bmatrix}^T \qquad \bar{F} = \begin{bmatrix} \bar{F}_1 & \bar{F}_2 & \bar{F}_3 \end{bmatrix}$$

$$\bar{F}_1 = (I - F_3CB)^{-1} F_1 \qquad \bar{F}_2 = (I - F_3CB)^{-1} F_2 \qquad \bar{F}_3 = (I - F_3CB)^{-1} F_3$$

The problem of PID controller design reduces to that of Static Output Feedback (SOF) [21-22] controller design for the following system:

$$\begin{cases} \dot{z} = \underline{A}z + \underline{B}u \\ \bar{y} = \bar{C}z \\ u = \bar{F}\bar{y} \end{cases} \quad (18)$$

Once $\bar{F}$ is found, the original PID gains can be recovered from

$$\begin{cases} F_3 = \bar{F}_3 (I + CB\bar{F}_3)^{-1} \\ F_2 = (I - F_3CB)\bar{F}_2 \\ F_1 = (I - F_3CB)\bar{F}_1 \end{cases} \quad (19)$$

The design problem of PID controllers under H_∞ performance specification is handled by first considering the system (11) rewritten as (Fig. 3):

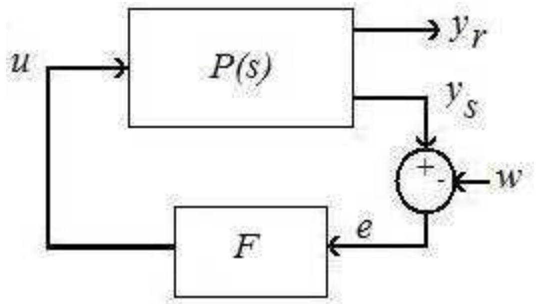

Fig. (3) Iterative PID block diagram.

$$P(s):\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ y_s=C_sx\\ y_r=C_r\,x+Du\end{cases} \tag{20}$$

where

x	state variables
u	control inputs
w	disturbance/reference inputs
y_s	sensed/measured outputs
yr	regulated/controlled outputs
A, B_1, B_2, C_s, and C_r	matrices with appropriate dimensions.

The static output feedback H_∞ control problem is to find a controller of the form

$$u=F\,y_s \tag{21}$$

such that the H_∞-norm of the closed-loop transfer function from w to y_r is stable and limited as follows:

$$\|G_{wy_r}\|_\infty<\gamma \tag{22}$$

Algorithm 1, shown in Appendix 2, is used to solve for the dynamics of the newly obtained SOF control system:

$$\begin{cases}\dot{z}=\bar{A}z+\bar{B}_1w+\bar{B}_2u\\ \bar{y}=\bar{C}_s\,z\\ \bar{y}_r=\bar{C}_r\,z+Du\\ u=\bar{F}\bar{y}_s\end{cases} \tag{23}$$

using

$$A=\bar{A},\quad B_1=\bar{B}_1,\quad B_2=\bar{B}_2\,,\quad C_s=\bar{C}_s\,,\quad C_r=\bar{C}_r\,,\quad F=\bar{F}$$

With

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \quad \bar{C}_s = \begin{bmatrix} C_s & 0 \end{bmatrix} \quad \bar{C}_r = \begin{bmatrix} C_r & 0 \end{bmatrix}$$

Thus, once the feedback matrices $\bar{F} = (\bar{F}_1, \bar{F}_2, \bar{F}_3)$ are obtained using *Algorithm 1* as applied to system (23), the original PID gains $F = (F_1, F_2, F_3)$ can be recovered from (19).

5. PID Design with H2

The design problem of PID controllers under H_2 performance specification is investigated, first, by studying the static output feedback (SOF) case and then extending the result to the PID case. As before, consider the system:

$$P(s) : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{24}$$

Assuming that A is stable then for the system closed-loop transfer function

$$G(s) = C(sI - A)^{-1}B + D \tag{25}$$

the classical result within Lyapunov approach gives

$$\| G \|_2^2 = Trace(B^T QB) \tag{26}$$

where Q is a solution of the following Lyapunov equation:

$$A^T Q + QA + C^T C = 0 \tag{27}$$

The dual form of H_2 norm formulation is:

$$\| G \|_2^2 = Trace(CPC^T) \tag{28}$$

where P is a solution of the following Lyapunov equation:

$$AP + PA^T + BB^T = 0 \tag{29}$$

The Static Output Feedback H_2 control (SOFH2) problem is to find a control of the form

$$u = Fy_s \tag{30}$$

such that the closed-loop transfer function, from w to y_r, is stable and

$$\|G_{wyr}\|_2 < \gamma \tag{31}$$

with $\gamma > 0$ and $\|.\|_2$ denotes the *2*-norm of the system transfer matrix.

The H_2-performance index, for system (20) rewritten as

$$
P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ y_s = C_s x \\ y_r = C_r x \end{cases}
$$

can be achieved by a SOF controller if the matrix inequalities:

$$
\begin{cases} trace(C_r P C_r^t) < \gamma^2 \\ AP + PA^T - PC_s^T C_s P + (B_2 F + PC_s^T)(B_2 F + PC_s^T)^T + B_1 B_1^T < O \\ P > 0 \end{cases} \tag{32}
$$

have solutions for (P,F).

An iterative LMI algorithm, *Algorithm 2*, for solving H_2-SOF control is developed in [21] and shown in *Appendix 3* where

$$
A = \bar{A}, \quad B_1 = \bar{B}_1, \quad B_2 = \bar{B}_2, \quad C_s = \bar{C}_s, \quad C_r = \bar{C}_r, \quad F = \bar{F}
$$

The PID design with H_2 specifications converts to a SOF control for the dynamics of the newly obtained system:

$$
\begin{cases} \dot{z} = \bar{A}z + \bar{B}_1 w + \bar{B}_2 u \\ \bar{y} = \bar{C}_s z \\ \bar{y}_r = \bar{C}_r z \\ u = \bar{F}\bar{y}_s \end{cases} \tag{33}
$$

Where

$$
\bar{A} = \begin{bmatrix} A & 0 \\ C_s & 0 \end{bmatrix} \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}
$$

$$
C_{s1} = \begin{bmatrix} C_s & 0 \end{bmatrix} \quad C_{s2} = \begin{bmatrix} 0 & I \end{bmatrix} \quad C_{s3} = \begin{bmatrix} C_s & 0 \end{bmatrix} \quad C_s = \begin{bmatrix} C_{s1}^T & C_{s2}^T & C_{s3}^T \end{bmatrix}^T \quad C_r = \begin{bmatrix} C_r & 0 \end{bmatrix}
$$

Thus, once the feedback matrices $\bar{F} = (\bar{F}_1, \bar{F}_2, \bar{F}_3)$ are obtained using *Algorithm 2* as applied to system (33), the original PID gains $F = (F_1, F_2, F_3)$ can be recovered from (19).

6. Maximum Output Control with PID

The design problem of a PID controller under the performance requirement that the system output y_r is smaller than a specified value σ when the input signal w is bounded, is known as Maximum Output Control (MOC) problem. To handle such problem, consider the system

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ y_s = C_s x \\ y_r = C_r x + Du \end{cases} \quad (34)$$

With x(0)=0. The Static Output Feedback Maximum Output Control (SOFMOC) problem is to find a control of the form

$$u = F y_s \quad (35)$$

such that the maximum regulated output $Y_{r,max}$, from w to y_r, of the closed-loop system, under the command input w, satisfies

$$Y_{r,max} = sup_{t \geq 0} \|y_r(t)\| \leq \sigma \qquad (\sigma > 0) \quad (36)$$

This is fulfilled if there exist matrices P>0 and F, and numbers $\tau_2 \geq 0$, η>0, such that the following linear matrix inequalities hold [21-22]:

$$\begin{bmatrix} P & (C_r + DFC_s)^T \\ (C_r + DFC_s) & \frac{\sigma^2}{\eta} I \end{bmatrix} > 0$$
$$\begin{bmatrix} \Sigma_3 & PB_1 \\ B_1^T P & -\tau_2 \eta I \end{bmatrix} < 0 \quad (37)$$

Where $\Sigma_3 = (A + B_2 FC_s)^T P + P(A + B_2 FC_s) + \tau_2 P$.

An iterative LMI algorithm (*Algorithm 3*) for solving SOFMOC is developed in [21-22] and shown in *Appendix 4*.

The PID design with MOC specifications converts to a SOFMOC for the dynamics of the newly obtained system

$$\begin{cases} \dot{z} = \bar{A}z + \bar{B}_1 w + \bar{B}_2 u \\ y = \bar{C}_s z \\ \bar{y}_r = \bar{C}_r z + Du \\ u = \bar{F} \bar{y}_s \end{cases} \quad (38)$$

So, *Algorithm 3* can be applied to (38) using

$A = \bar{A}, \quad B_1 = \bar{B}_1, \quad B_2 = \bar{B}_2, \quad C_s = \bar{C}_s, \quad C_r = \bar{C}_r, \quad F = \bar{F}$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C_s & 0 \end{bmatrix} \qquad \bar{B}1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \qquad \bar{B}2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix}$$

$$C_{s1} = [C_s \quad 0], \quad C_{s2} = [0 \quad I], \quad C_{s3} = [C_s \quad 0], \quad C_s = \begin{bmatrix} C_{s1}^T & C_{s2}^T & C_{s3}^T \end{bmatrix}^T, \quad C_r = [C_r \quad 0]$$

As before, to recover the original PID gains $F = (F_1, F_2, F_3)$ from the feedback matrices $\bar{F} = (\bar{F}_1, \bar{F}_2, \bar{F}_3)$, the relations in (19) can be applied.

7. Simulation Results

To demonstrate the effectiveness of a PID controller designed as *Hi*, *H2* and *Max* while driving the plant, several tests are carried out and the results are presented and compared with those of the classical robust controller *ROB*. The simulation results are obtained using MATLAB package and LMI Toolbox.

A.) Parameters of the robust controller (ROB):

Initial condition (operating point) for the nonlinear system:

$$x_0 = [0.775 \quad 0 \quad 1.434 \quad -0.0016 \quad 0.8 \quad 0.8]^T$$

Plant *P(s):*

$$P(s): \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_z x + D_{z1} w + D_{z2} u \\ y = C_y x + D_{y1} w + D_{y2} u \end{cases}$$

$$C_z = C_1 = +C, \quad C_y = C_2 = -C, \quad D = \begin{bmatrix} D_{z1} & D_{z2} \\ D_{y1} & D_{y2} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -37.6 & -1.5 & -26 & 0 & 0 & 29.6 \\ -0.3 & 0 & -0.56 & 314 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 1.25 & -1.25 \end{bmatrix},$$

$$B_1 = 0_{6x2}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.1 \\ 18.89 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix}1.27 & 0 & 0.88 & 0 & 0 & 0\\ 0.03 & 0 & 0.53 & 0 & 0 & 0\end{bmatrix},\qquad D=0_{2x2}$$ Controller $K(s)$:

$$A_k=\begin{bmatrix}-72 & 19.3 & 97 & -124 & 182 & -705\\ -33 & -46 & -40.5 & -34 & 368 & -1658\\ 95 & 65 & -179 & -77 & 74 & -104\\ -1.9 & 36 & -9 & -99 & 117 & -378\\ 38 & -252 & 79 & 672 & -881 & 2808\\ -219 & 1279 & -299 & -3476 & 4503 & -14527\end{bmatrix},\quad B_k=\begin{bmatrix}212 & 126\\ -24 & -14\\ 131 & 76\\ 172 & 88\\ -1155 & -725\\ 6009 & 3608\end{bmatrix},$$

$$C_k=\begin{bmatrix}-4 & -1.6 & 7.9 & 0.35 & -1.4 & 0.9\\ 407 & 360 & 35 & 351 & -2887 & 13030\end{bmatrix},\quad D_k=0$$

Desired H∞-norm: $\gamma=100$

Optimum H∞-norm: $\gamma_{opt}=7.8603$

Closed-loop eigenvalues: λ_{CL} = [-15370, -103±436i, -229, -10, -4.7, -2.2 ±2.7i, -1.3± 2.8i, -0.63, -1]^T

B.) Parameters of H∞-PID controller (Hi):

The obtained PID gains are:

$$\bar{F}_1=\begin{bmatrix}-422 & 2029\\ 354 & -1720\end{bmatrix}\quad \bar{F}_2=\begin{bmatrix}-24 & 803\\ 14 & -681\end{bmatrix}\quad \bar{F}_3=\begin{bmatrix}-218 & 435\\ 185 & -368\end{bmatrix}$$

$$\bar{F}=\left[\bar{F}_1\quad \bar{F}_2\quad \bar{F}_3\right]$$

Riccati starting matrix: $Q_0=10I_{8x8}$

Desired dominant eigenvalue: $\alpha_{opt}=0$

Obtained dominant eigenvalue: $\alpha_{opt}=-0.77$

Closed-loop eigenvalues: λ_{CL} = [-998-6.4 ± 15.8i-6 ± 5.6i-0.43-1.87-4.1]^T

C.) Parameters of H2-PID controller (H2):

$$\bar{F}_1=\begin{bmatrix}+0.14 & +0.34\\ -1.55 & -34.2\end{bmatrix}\quad \bar{F}_2=\begin{bmatrix}-0.21 & +0.22\\ -3.10 & -29.3\end{bmatrix}\quad \bar{F}_3=\begin{bmatrix}+0.24 & +0.19\\ +2.86 & -17.9\end{bmatrix}$$

$$\bar{F}=\left[\bar{F}_1\quad \bar{F}_2\quad \bar{F}_3\right]$$

Initial Riccati matrix: $Q_0=10*I_{8x8}$

Closed-loop eigenvalues: λ_{CL} =[-225-4.4 ± 4i-1.7 ± 1.97i-1±0.74-1.16]^T

D.) Parameters of MOC-PID controller:

$\eta=100$

$\sigma=50$

$$\bar{F}_1 = \begin{bmatrix} -0.021 & +1.3 \\ -47.3 & -152 \end{bmatrix} \quad \bar{F}_2 = \begin{bmatrix} -0.32 & +0.53 \\ -27 & -52 \end{bmatrix} \quad \bar{F}_3 = \begin{bmatrix} -0.035 & +0.51 \\ +3.75 & -88 \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} \bar{F}_1 & \bar{F}_2 & \bar{F}_3 \end{bmatrix}$$

Initial Riccati matrix: $Q0=I_{8x8}$

Closed-loop eigenvalues: λ_{CL} =[-1362-9.94-1.1 ±5.9i-1.1 ± 1.04i-0.48-0.81 $]^T$

Test 1: Step-response

To test the effectiveness of the system equipped with each of the named three LMI-based iterative multivariable PID namely; PID design using H_∞-norm (*Hi*), PID design using H_2-norm (*H2*), PID design with Maximum Output Control (*Max*), and the LMI-based robust output feedback controller using H_∞-norm (*ROB*), an increase (at t=0 s) then a decrease (at t=15 s) by 5% in both P_{ref} and V_{ref} is applied. The time responses of the exciter input voltage U_e, the governor valve position U_g, the output active power P_t, and the terminal voltage V_t, are shown, respectively, in Fig. (4). Best performance is characterized by lower or no over/undershoots, less or no oscillations, short rise and settling times. Based on this, *Hi* shows the best response whereas *ROB* shows the worse response with higher overshoots. For V_t response, *H2* shows the best response whereas *Max* shows the worse one with longer settling time.

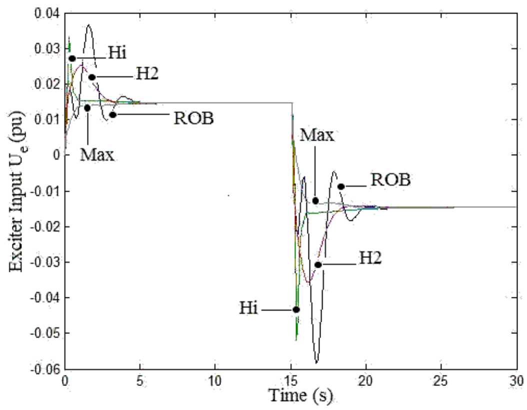

Fig. (4). Step-response following P_{ref}=V_{ref} =5% (a) Exciter Input U_e

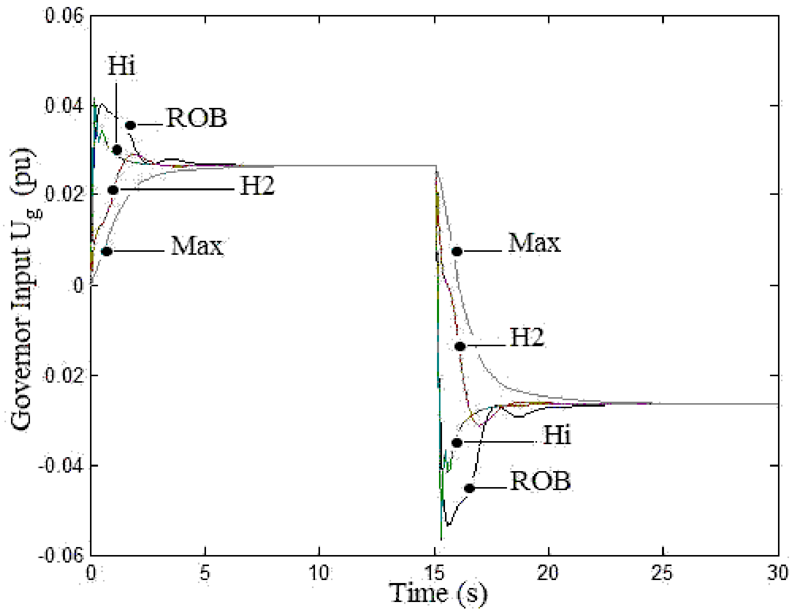

Fig. (4.b). Governor input U_g

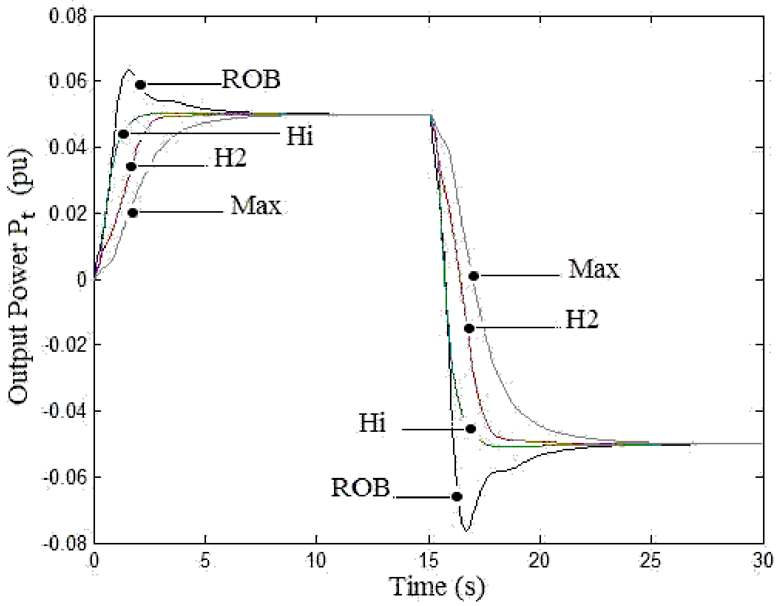

Fig. (4.c). Power output P_t

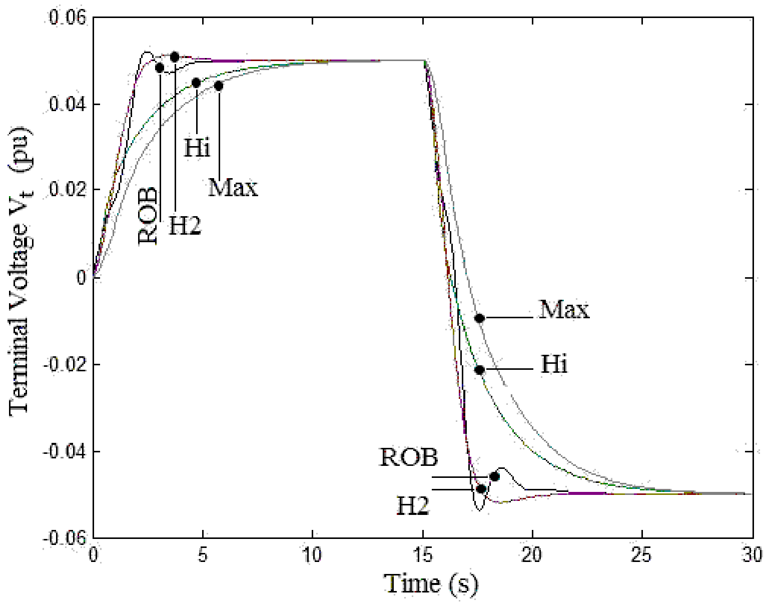

Fig. (4.d). Terminal voltage V_t

Test 2: Tracking-response

To test the effectiveness of the system to tracking the reference control values, the simulation period is divided into 4 regions where the reference values of the controlled variables (V_{ref} and P_{ref}) increase linearly, then remain steady, then a linear decrease, and finally return to nominal values. The time responses of the exciter input voltage (voltage control effort) U_e, the governor valve position (governor control effort) U_g, the output active power P_t, and the terminal voltage V_t, are shown, respectively, in Fig.(5). For P_t-response, *Hi* shows the best response whereas *H2* shows the worse with longer settling time. For V_t response, *Max* shows the best response whereas *ROB* shows the worse one with longer settling time.

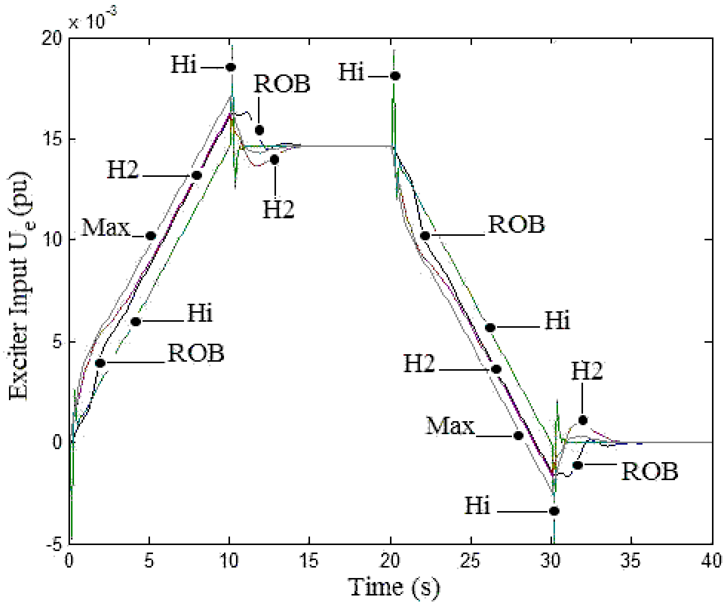

Fig. (5). System response due to reference tracking (*test 2*) .(a) Exciter Input U_e

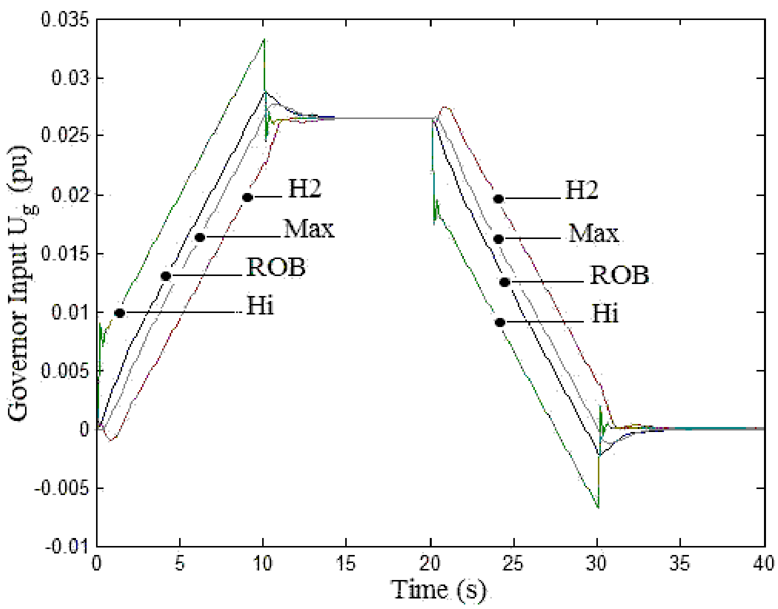

Fig. (5.b). Governor input U_g

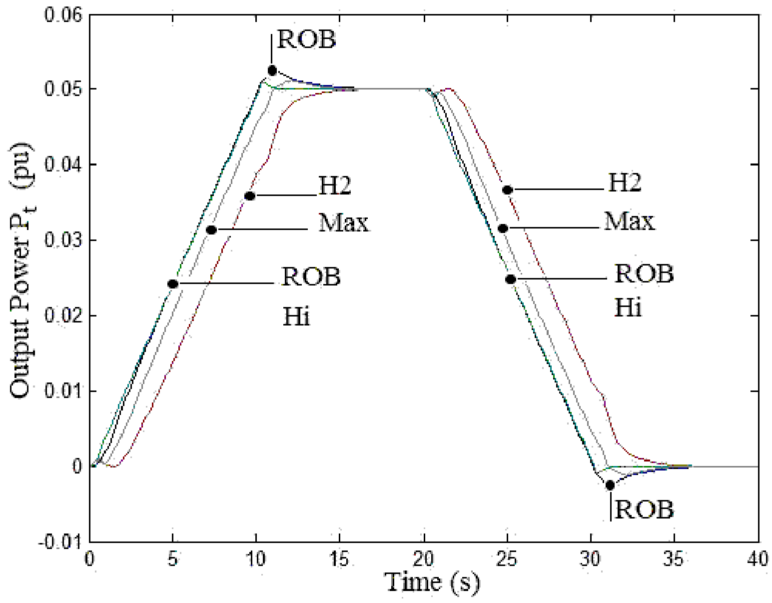

Fig. (5.c). Power output P_t

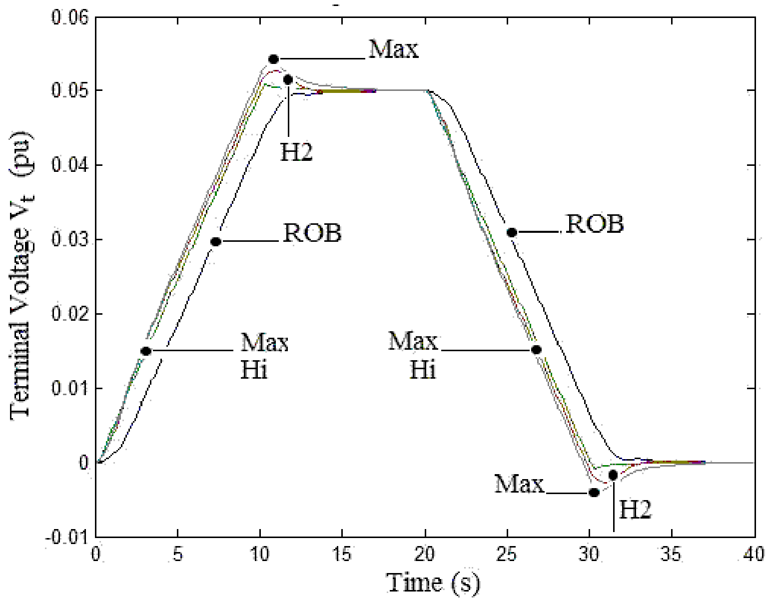

Fig. (5.d). Terminal voltage V_t

Test 3: Parameters Variation

To test the robustness to parameters change, an increase by 50% in the inertia constant H and in the damping torque coefficient T_d are applied. Fig.(6) shows the system response following a step change by 5% then -5% in V_{ref} and T_{ref} with the system experiencing the described parameters change and using the controller gains found for the normal case.

The time responses of the exciter input voltage (voltage control effort) U_e, the governor valve position (governor control effort) U_g, the output active power P_t, and the terminal voltage V_t, are shown, respectively, in Fig.(6). For P_t-response, *Hi* shows the best response whereas *ROB* shows the worse one with large over/undershoots. The other two, exhibit relatively larger rising and settling times. For V_t response, *H2* shows the best response whereas *Max* shows the worse one with longer settling time.

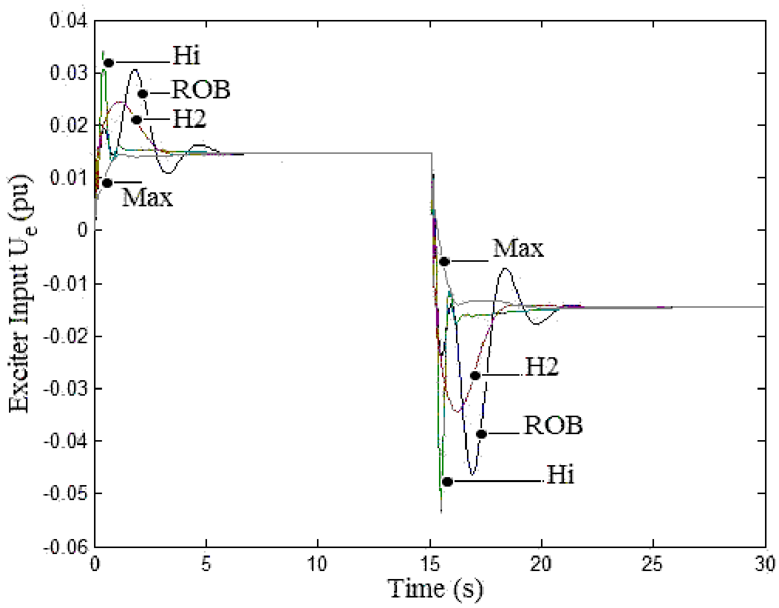

Fig. (6). System response with parameters change (*test 3*). (a) Exciter Input U_e

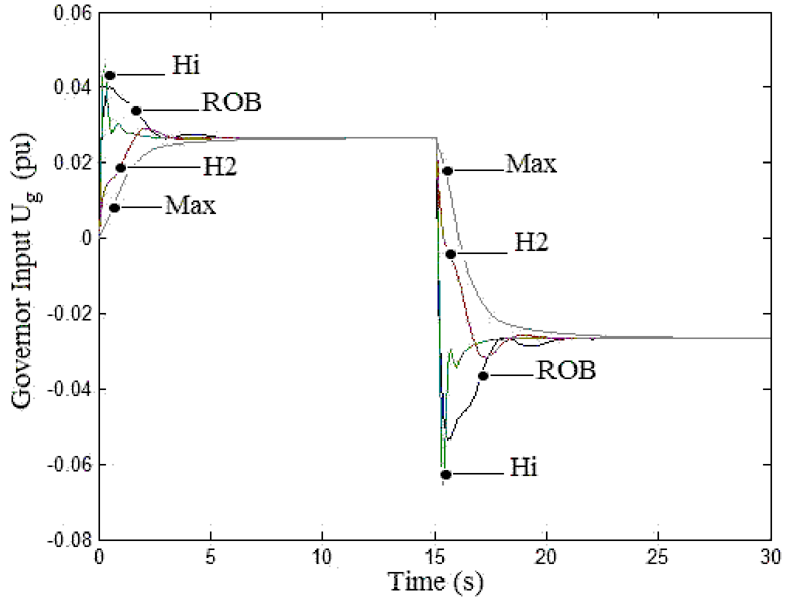

Fig. (6.b). Governor input U_g

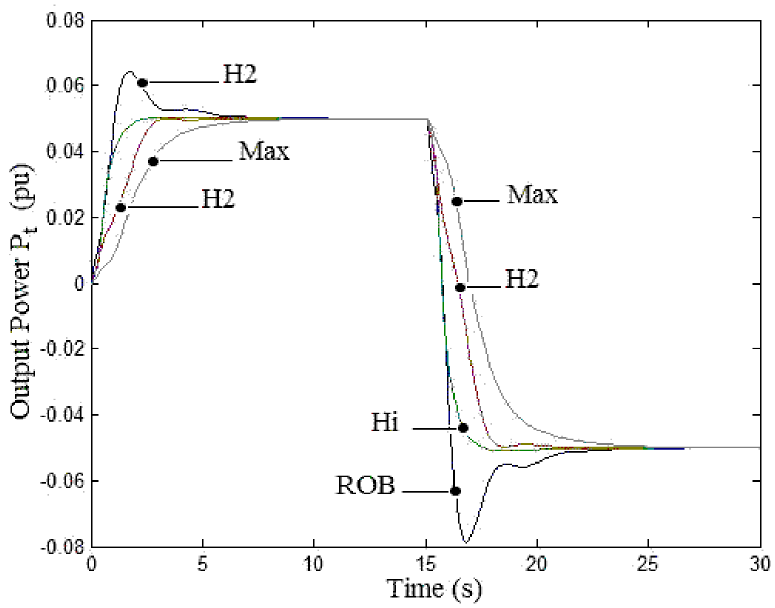

Fig. (6.c). Power output P_t

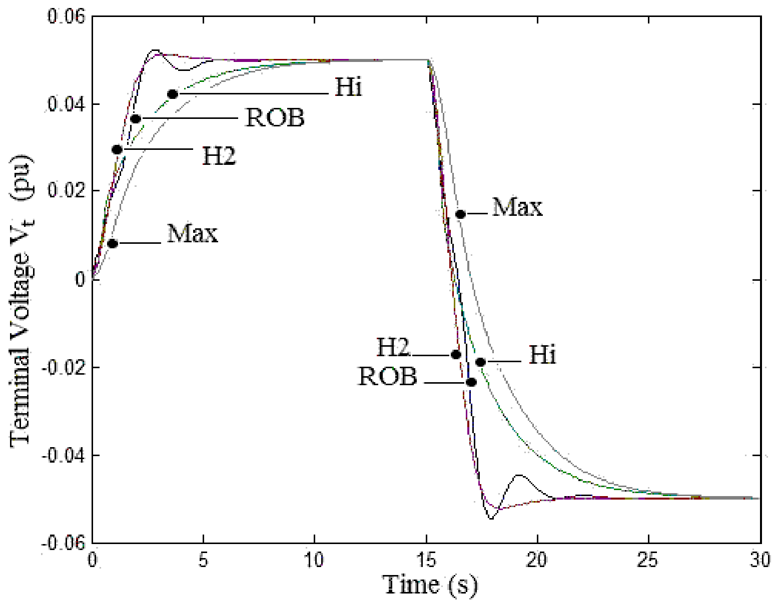

Fig. (6.d). Terminal voltage V_t

8. Conclusion

Four controllers, the first, a robust H_∞-LMI based output feedback (ROB) and the other three LMI-based iterative multivariable PID controllers namely; PID design with H_∞-specifications (*Hi*), PID design with H_2-specifications (*H2*), PID design with maximum output control (*Max*), were designed for a sample power system comprising a steam turbine driving a synchronous generator connected to an infinite bus via a step-up transformer and a transmission line. Several tests were applied to allow for a comparative study between the performances of the proposed controllers. The quality of the controller response is selected through its performance that is characterized by lower or no over/under shoots, less or no oscillations, short rise and settling times.

From the simulation results, it is clear that in all cases, PID exhibits better performance than the classical robust control (*ROB*). *Hi* shows the best responses for P_t in all the tests done whereas, for V_t, *H2* and *Max* present the best response for all tests done. ROB has another inconvenient that is its high order which is equal to the plant model order, thus it is more complicated in its implementation.

As an extension, the performance of the PID via multi-objective and poles placement, the extension to a multimachine power system, and the inclusion of the nonlinear features inherent in the system, will be considered in the future. Moreover, more tests should be done, and diverse controller parameters should be varied to extract all features of each of the cited controllers.

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Appendix

APPENDIX 1: SYSTEM PARAMETERS

MVA	37.5	x_t	0.345 pu	K_2	-0.9218
MW	30	x_l	0.125 pu	K_3	-0.5609
p.f.	0.8 lagging	e	1 pu	K_4	0.4224
kV	11.8	τ_e	0.1 s	K_5	0.7983
r/min	3000	τ_g	0.1 s	K_6	0.5905
x_d	2 pu	τ_b	0.5 s	K_7	0.3650
x_q	1.86 pu	K_v	1.889	K_8	-39.559
x_{ad}	1.86 pu	K_e	0.01	K_9	-27.427
x_{fd}	2 pu	vd	0.5586	K_{10}	-0.2955
R_{fd}	0.00107 pu	vq	1.1076	K_{11}	1.268
H	5.3 MWs/MVA	V_t	1.2405	K_{12}	0.8791
T_d	0.05 s	K_1	1.2564	K_{13}	0.0287
				K_{14}	0.52726

APPENDIX 2: ALGORITHM 1 (HI)

<u>*Step 0:*</u> Form the system state space realization: (A, B_1, B_2 ,C_s ,C_r , D) and select the performance index γ

<u>*Step 1:*</u> Choose $Q_0 > 0$ and solve P for the Riccati equation:

$$A^T P + PA - PB_2 B_2^T P + Q_0 = O, \quad P > 0$$

Set i= 1 and $X = P$

<u>*Step 2:*</u> Solve the following optimization problem for P, F and α_i.

OP1: Minimize α_i subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_1 & PB_1 & (C_r + DFC_s)^T & (B_2^T P + FC_s)^T \\ B_1^T P & -\gamma & 0 & 0 \\ C_r + DFC_s & 0 & -I & 0 \\ B_2^T P + FC_s & 0 & 0 & -I \end{bmatrix} < 0 \qquad \text{(A1)}$$

$$P > 0$$

Where

$$\Sigma_1 = A^T P + PA - XB_2 B_2^T P - PB_2 B_2^T X + XB_2 B_2^T X - \alpha P$$

Denote by α^* the minimized value of α.

Step 3: If $\alpha^* \leq 0$, the matrix pair (P,F) solves the problem. Stop. Otherwise go to Step 4.

Step 4: Solve the following optimization problem for P and F.

OP2: Minimize *trace*(P) subject to LMI constraints (A1) with $\alpha = \alpha^*$. Denote by P* the optimal P.

Step 5: If $\| XB - P^* B \| < \varepsilon$. where ε is a prescribed tolerance, go to Step 6; Otherwise set i= i + 1, $X = P^*$, go to Step 2.

Step 6: It cannot be decided by this algorithm whether the problem is solvable. Stop.

Appendix 3: Algorithm 2 (H2)

Step 0: Form the system state space realization: (A, B_1, B_2 ,C_s ,C_r) and select the performance index γ

Step 1: Choose $Q_0 > 0$ and solve P for the Riccati equation:

$$AP + PA^T - PC_S^T C_S P + Q_0 = O, \quad P > 0$$

Set i= 1 and $X = P$

Step 2: Solve the following optimization problem for P_i, F and α_i.

OP1: Minimize α subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_2 & B_2 F + PC_s^T \\ (B_2 F + PC_s^T)^T & -I \end{bmatrix} < 0$$

$$trace\,(C_r P C_r^T) < \gamma^2 \qquad \text{(A2)}$$

$$P > 0$$

Where

$$\Sigma_2 = AP + PA^T + B_1 B_1^T - XC_s^T C_s P - PC_s^T C_s X + XC_s^T C_s X - \alpha .P$$

Denote by α^* the minimized value of α.

Step 3: If $\alpha^*\leq 0$, the matrix pair (P,F) solves the problem. Stop. Otherwise go to Step 4.
Step 4: Solve the following optimization problem for P and F.

OP2: Minimize *trace*(P) subject to LMI constraints (A2) with $\alpha = \alpha^*$. Denote by P* the optimal P.

Step 5: If $\|XB-P^*B\|<\varepsilon$. where ε is a prescribed tolerance, go to Step 6; otherwise set i= i+1, $X=P^*$, and go to Step 2.
Step 6: It cannot be decided by this algorithm whether the problem is solvable. Stop.

Appendix 4: Algorithm 3 (Max)

Step 0: Let the system state space realization (A, B_1, B_2, C_s, C_r, D), a performance index σ, and a given number $\eta>0$ be given
Step 1: Choose $Q_0 > 0$ and solve P for the Riccati equation: $A^T P + PA - PB_2 B_2^T P + Q_0 = O, \quad P > 0$

Set i= 1 and $X = P$

Step 2: Solve the following optimization problem for P, F and α.

OP1: Minimize α subject to the following LMI constraints

$$\begin{bmatrix} \Sigma_4 & PB_1 & (B_2^T P + FC_s)^T \\ B_1^T P & -\tau_2 \eta I & 0 \\ B_2^T P + FC_s & 0 & -I \end{bmatrix} < 0 \qquad \text{(A3)}$$

$$\begin{bmatrix} P & (C_r + DFC_s)^T \\ (C_r + DFC_s) & \dfrac{\sigma^2}{\eta} I \end{bmatrix} > 0$$

$$P > 0$$

Where

$$\Sigma_4 = A^T P + PA + XB_2 B_2^T P - PB_2 B_2^T X + XB_2 B_2^T X + \tau_2 P - \alpha P.$$

Denote by α^* the minimized value of α.

Step 3: If $\alpha^*\leq 0$, the matrix F solves the problem. Stop. Otherwise go to Step 4.
Step 4: Solve the following optimization problem for P, F.

OP2: Minimize *trace*(P) subject to LMI constraints (A3) with $\alpha = \alpha^*$. Denote by P* the optimal P.

Step 5: If $\|XB-P^*B\|<\varepsilon$. where ε is a prescribed tolerance, go to Step 6; otherwise set i= i+1, $X=P^*$,

$$\tau_2 = \sqrt{\frac{trace(P^* B_1 B_1^T P^*)}{\eta\ tr(P^*)}}$$

and go to Step 2.

Step 6: It cannot be decided by this algorithm whether the SOFMOC problem is solvable. Stop.