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Economic Order Quantity with Stochastic Demand, Imperfect Quality, and Inspection Errors

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Abstract. In this paper, EOQ models with stochastic demand for items with imperfect quality are developed for two cases; with and without replacement of nonconforming items, considering 100% inspection policy with the assumption that an imperfect inspection is performed. An order is considered to be placed from a supplier when the inventory level reaches the reorder point. When the order is received, an inspector will inspect the whole lot with the consideration that Type I or Type II inspection errors could be committed by the inspector during the inspection process. The probability of misclassification errors is assumed to be known. The fraction nonconforming is assumed to be a random variable following a known distribution. The objective of this research is to determine the optimal order quantity and reorder point such that the total cost is minimized. A solution for determining the order quantity and reorder point is proposed. Numerical examples with some sensitivity analysis for important model parameters are provided for the proposed model. Possible future extensions to the presented model are presented in the conclusion.

Keywords: EOQ; Stochastic demand; Imperfect quality; Inspection errors

List of Symbols

D demand rate (unit/unit time)
x demand during lead time
$f(x)$ probability density function of x
A fixed ordering cost
h holding cost per unit per unit time
π shortage cost per unit
$S(x)$ shortage quantity per cycle
$\bar{S}(x)$ expected shortage quantity per cycle
p probability that an item is defective
e_1 probability of Type I error (classifying a nondefective item as a defective)
e_2 probability of Type II error (classifying a defective item as a nondefective)
$f(p)$ probability density function of p
$f(e_1)$ probability density function of e_1
$f(e_2)$ probability density function of e_2
C_i inspection cost per unit
C_r rectifying cost per unit
C_d cost of accepting a defective item
C_n cost of rejecting a nondefective item
Q lot size
r reorder point
T cycle length

1. Introduction

In the classic economic order quantity (EOQ) model, many simplifying assumptions are made when developing a closed form solution for the optimal order quantity. One particular assumption of relevance to the current study is that the items produced are all of perfect quality. Random yield production processes have gained considerable interest by researchers for theoretical and practical purposes. The production lot may contain a number of defective items, which could result from weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit [1]. In models that assume imperfect quality items, it is assumed that the inspection process for detecting the defective items in a lot is error-free. However, the inspection process is subjected to inspection errors, Type I and Type II errors, which might be committed by the inspector, e.g. inaccuracy in inventory records [2], which their presence may seriously affect the product quality. Hence, there is a need to determine the optimal order quantity for imperfect-quality items with the consideration of imperfect inspection [3].

In this paper, EOQ models with stochastic demand for items with imperfect quality have been proposed. The proposed models have the following features: The order quantity and reorder point are the decision variables; models are developed for the case when nonconforming items are replaced and also for the case where nonconforming items are discarded without replacement, and 100% inspection policy is adopted with the assumption of an imperfect inspection is performed; that is, Type I and Type II errors might occur during the inspection process.

The remainder of this paper is organized as follows. A literature review is presented in section 2. Problem definition, notation and assumptions are presented in Section 3. Joint inventory inspection model with and without replacement of nonconforming items are developed in Section 4. Section 5 contains models analysis for determining optimal solutions. In Section 6, numerical examples are provided for the proposed solution. Finally, conclusion and possible future works are stated in Section 7.

2. Literature Review

A literature review is presented in this section for a number of literature models that have dealt with the relationship between ordering quantity and quality control. The literature is classified based on the consideration of inspection errors.

2.1 Considering inspection errors

Reference [4] incorporated Type I and Type II inspection errors into an economic manufacturing quantity EMQ model under the imperfect production system and derived the expected total cost with the objective of determining the optimal production cycle length and optimal inspection number. Reference [5] proposed multi-stage lot sizing models for imperfect production processes and considering the effect of imperfect quality on lot sizing decisions and effect of inspection errors. Reference [6] developed a production inventory model that takes into account the effect of imperfect production processes, preventive maintenance and inspection errors with the objective of finding the optimal production quantity where shortages are not allowed. They considered 100% inspection policy where the defective items detected are discarded. Also, Reference [7] proposed a profit-maximizing economic production quantity model with no shortages that incorporates imperfect production quality and imperfect inspection adopting 100% inspection policy and considering rework and salvage for defective items detected. Reference [8] considered a simple single-vendor single-buyer supply chain system in which products are received with defective quality; and 100% inspection process is performed with possible inspection errors considering that defective items found will be replaced. He developed a cost model for the supply chain system in which shortages are not allowed and with the objective of determining the optimal number of shipments as well as the size of each shipment. Reference [9] investigated the integration of the acquisition of input materials, material inspection and production planning, where type I and type II inspection errors are allowed, and the unit acquisition cost is dependent on the average quality level. They aimed to find an optimal purchase lot size, input quality level and the associated inspection policy that minimize the total cost per item with the assumption that shortages are not allowed and the defective items found will be replaced. Reference [10] proposed a cost-minimizing EOQ model that incorporates imperfect production quality, inspection errors, shortages backordered, and quantity discounts considering 100% inspection processes with possible inspection errors. Reference [3] determined an optimal production/order quantity where shortages are not allowed. They considered 100% inspection policy with the assumption of imperfect inspection process is performed and detected defective items would be salvaged as a single batch and sold at a lower price. Reference [11] developed an EOQ model when items are of perfect and imperfect quality and a single acceptance sampling plan with destructive testing and inspection errors is adopted.

2.2 Not considering inspection errors

Reference [12] studied a joint lot sizing and inspection policy under an EOQ model where a random proportion of units are defective and can be discovered only through costly inspections. They developed a model for finding optimal lot size and fraction to inspect with the consideration that defective items found will be replaced. References [13] and [14] assumed that the arrived lot may contain some defective items, and they adopted a sub-lot inspection policy. Also they assumed that uninspected defective items which were sold can be returned which resulted in an extra treatment cost for the vendor. They considered a continuous review inventory model with a mixture of backorders and lost sales in which the order quantity, reorder point, and lead time are the decision variables. References [15] and [16] studied a production/inventory situation where items, received or produced, are not of perfect quality. They developed an EOQ/EPQ model for determining the optimal production/order quantity with the assumption that shortages are not allowed and poor-quality items will be sold as a single batch at discounted priced after the end of the 100% inspection process. Reference [17] provided a framework to integrate lower pricing, rework and reject situations into a single EPQ model considering shortages are not allowed and 100% inspection is performed in order to identify the amount of good quality items, imperfect quality items and defective items in each lot with the assumption that defective items could be used in another production situation or sold at a lower price. References [18] and [19] developed integrated inventory inspection models with and without replacement of nonconforming items where inspection policies include no-inspection, sampling inspection, and 100% inspection considering deterministic and stochastic demand, respectively, and shortages are backordered. They proposed a solution procedure for determining the operating policies for inventory and inspection consisting of the order quantity reorder point, sample size, and acceptance number. References [20] and [21] developed an EOQ model for which each ordered lot contains some defective items and shortages backordered with the assumption that 100% inspection process is performed for each lot and the found defective items are either sold at a lower price or discarded. Reference [22] considered a production/inventory system with the assumptions that shortages are backordered, each lot contains a random proportion of defective units, the purchaser conducts a 100% inspection to identify the acceptable items, and the detected imperfect items are either sold in a secondary market, as a single batch and at a lower price, or reworked. Reference [23] developed an inventory model for items with imperfect quality and quantity discounts; the defectives are screened out by a 100% inspection for each shipment and sold in a batch by the end of inspection at the last shipment of each cycle. Reference [24] studied an inventory system with the assumptions that demand is satisfied by recovered and new purchased items, the shortages are not allowed, returned items by customers are kept in recoverable inventory until the start of a combined process of inspection and recovery, and the recovered items are as-good-as new. However, if recovered items do not qualify to be classified as remanufactured, they will be sold in a secondary market at a reduced price.

The models in the literature do not consider stochastic demand and errors in inspection in a single treatment. This article will bridge this gap in the literature by developing models that combine stochastic demand and inspection errors.

3. Problem Definition

Orders of size Q are placed from a supplier when the stock drops down to the reorder level r Due to the uncertainty in demand during lead time, there are chances of shortages if demand is underestimated and high holding costs if demand is overestimated. When shortages occur, they are backordered. The incoming quality of received lots is stochastic in nature following a known distribution and each lot contains a fixed proportion p of defective items. When a lot is received, an inspector will inspect the whole lot; 100% inspection policy is adopted, with a fixed rate of misclassification; a proportion e_1 of nondefective items are classified to be defective and a proportion of e_2 defective items are classified to be nondefective. It is assumed that the probability density functions $f(p)$, $f(e_1)$, and $f(e_2)$ are known. There are two scenarios regarding the nonconforming items observed during inspection process; they are either all replaced or all discarded. Both of these cases are discussed here as non-replacement case and replacement case of nonconforming items. In each case, the costs considered are: the setup cost, inventory holding cost, screening cost (inspection cost and misclassification cost resulted due to imperfect inspection), rectifying cost (in the case when nonconforming items are replaced), and shortage cost. The aim is to determine the optimal ordering quantity Q and reorder level r such that the total expected cost is minimized.

Demand during the lead time is assumed to be stochastic and shortages are backordered. Moreover, the following assumptions are made regarding the inspection process: the fraction of nonconforming in the incoming lot is stochastic and follows a known distribution. An imperfect inspection process is performed; and the inspection time is negligible. Two cases are considered for the nonconforming items detected during the inspection process: they are either all discarded without replacement or all replaced. In case that the found nonconforming items are replaced, the replacements are delivered within the same inventory cycle.

4. Joint Inventory Inspection Models

In this section, a joint inventory inspection model is developed considering two cases: (1) nonconforming items found during inspection are discarded and (2) nonconforming items are replaced with good items. Since x, z, y, p, e_1 and e_2 are random variables, their expected values are used in determining the total expected annual cost for both cases.

4.1 Model without replacement of nonconforming items

In this case, the total cost includes the setup cost, inventory holding cost, screening cost (inspection cost and misclassification cost) and shortage cost. The setup cost is A for each time an order is placed. The inventory holding cost per cycle is:

$$HC = hT\left(\frac{(1-E[p])(1-E[e_1])Q}{2} + r - E[x]\right) \quad (1)$$

The screening cost per cycle is the sum of inspection cost and misclassification cost, which is resulted due to inspection Type I and Type II errors, and can be written as:

$$SC = C_i Q + C_n(1-E[p])E[e_1]Q + C_d E[p]E[e_2]Q \quad (2)$$

Where $[C_n(1-E[p])E[e_1]Q]$ is the cost resulted due to Type I error; and $[C_d E[p]E[e_2]Q]$ is the cost resulted due to Type II error. The expected shortage quantity per cycle is:

$$\bar{S}(x) = \int_0^{\infty} S(x)f(x)\,dx = \int_r^{\infty} (x-r)f(x)dx \tag{3}$$

Hence, the shortage cost per cycle will be $\pi\bar{S}(x)$.Therefore the total expected cost per cycle will be:

$$E[TC_1(Q,r)] = A + hE[T]\left(\frac{(1-E[p])(1-E[e_1])Q}{2} + r - E[x]\right) + C_iQ + C_n(1-E[p])E[e_1]Q + C_dE[p]E[e_2]Q + \pi\bar{S}(x) \tag{4}$$

The expected cycle length is equal to:

$$E[T] = \frac{(1-E[p])(1-E[e_1])Q}{D} \tag{5}$$

Therefore, by dividing the total expected cost per cycle (Eq. 4) by the expected cycle length (Eq. 5), the total expected annual cost will be:

$$E[TAC_1(Q,r)] = \frac{DA}{(1-E[p])(1-E[e_1])Q} + h\left(\frac{(1-E[p])(1-E[e_1])Q}{2} + r - E[x]\right) + \frac{DC_i}{(1-E[p])(1-E[e_1])} + \frac{C_nE[e_1]D}{(1-E[e_1])} + \frac{C_dE[p]E[e_2]D}{(1-E[p])(1-E[e_1])} + \frac{\pi D\bar{S}(x)}{(1-E[p])(1-E[e_1])Q} \tag{6}$$

4.2 Model with replacement of nonconforming items

In this case, the total cost includes the same costs considered in the previous model except for the inventory holding cost as the average level of the typical inventory cycle for this model differs from the previous one. So the inventory holding cost per cycle for this model is:

$$HC = hT\left(\frac{Q}{2} + r - E[x]\right) \tag{7}$$

Also, as the defective items are assumed to be rectified in this model, so the rectifying cost will be added, which is equal to $C_rE[p]Q$ per cycle.Therefore the total expected cost per cycle will be:

$$E[TC_2(Q,r)] = A + hE[T]\left(\frac{Q}{2} + r - E[x]\right) + C_iQ + C_n(1-E[p])E[e_1]Q + C_dE[p]E[e_2]Q + \pi\bar{S}(x) + C_rE[p]Q \tag{8}$$

Since all the nonconforming items will be replaced, so the expected cycle length will be:

$$E[T] = \frac{Q}{D} \tag{9}$$

Therefore, by dividing the total expected cost per cycle (Eq. 8) by the expected cycle length (Eq. 9), the total expected annual cost will be:

$$E[TAC_2(Q,r)] = \frac{DA}{Q} + h\left(\frac{Q}{2} + r - E[x]\right) + DC_i + C_n(1-E[p])E[e_1]D + C_d E[p]E[e_2]D + \frac{\pi D\bar{S}(x)}{Q} + C_r E[p]D \quad (10)$$

5. Models Analysis

We need to find the optimal order quantity Q and optimal reorder point r. So, the expressions for optimal order quantity Q and reorder point r are determined in this section for both models developed in the previous section.

5.1 Model without replacement of nonconforming items

In order to find the optimal Q, we find the first partial derivative of the total expected annual cost (Eq. 6) with respect to Q, set the derivative equal to zero, and solve for Q as follows:

$$\frac{dE[TAC_1(Q,r)]}{dQ} = -\frac{(1-E[p])(1-E[e_1])DA}{(1-E[p])^2(1-E[e_1])^2Q^2} + \frac{h(1-E[p])(1-E[e_1])}{2} - \frac{(1-E[p])(1-E[e_1])\pi D\bar{S}(x)}{(1-E[p])^2(1-E[e_1])^2Q^2} = 0$$

Hence, the optimal value of Q will be:

$$Q^* = \frac{1}{(1-E[p])(1-E[e_1])}\sqrt{\frac{2D(A+\pi\bar{S}(x))}{h}} \quad (11)$$

Following the same way of determining Q^*, we can find the expression for optimal reorder point as follows:

$$\frac{dE[TAC_1(Q,r)]}{dr} = h - \frac{\pi D}{(1-E[p])(1-E[e_1])Q}\int_r^{\infty} f(x)\,dx = 0$$

Hence, the optimal value of r will be:

$$\int_{r^*}^{\infty} f(x)\,dx = \frac{hQ^*(1-E[p])(1-E[e_1])}{\pi D} \quad (12)$$

5.2 Model with replacement of nonconforming items

Following the same steps applied on the previous model, we can find the optimal value expressions for Q and r for the total expected annual cost (Eq. 10) as follows:

$$\frac{dE[TAC_2(Q,r)]}{dQ} = -\frac{DA}{Q^2} + \frac{h}{2} - \frac{\pi D \bar{S}(x)}{Q^2} = 0$$

Hence, the optimal value of Q will be:

$$Q^* = \sqrt{\frac{2D(A + \pi \bar{S}(x))}{h}} \tag{13}$$

The first partial of Eq. 10 with respect to r is:

$$\frac{dE[TAC_2(Q,r)]}{dr} = h - \frac{\pi D}{Q} \int_r^\infty f(x)\,dx = 0$$

Hence, the optimal value of r will be:

$$\int_{r^*}^\infty f(x)\,dx = \frac{hQ^*}{\pi D} \tag{14}$$

Note that Eq. 12 and Eq. 14 describe the relationship between the reorder point r^* and the cost parameters of the model for each case. If the shortage cost π is greater than holding cost h, it is better to carry more inventory than to risk a shortage and vice versa.

6. Results and Discussion

In this section, some examples are solved using the proposed solutions in the previous section. The purpose is to illustrate the proposed solutions and conduct some sensitivity analysis for important model parameters.

The data of the examples are taken from [19]: $A = 75$, $D = 50{,}000$, $h = 5$, $C_i = 1.5$, $C_n = 15$, $C_d = 30$, $C_r = 5$, $\pi = 20$. Demand during lead time follows uniform distribution between 0 and 50 with mean 25.

$$f(x) = \begin{cases} \frac{1}{50} & 0 \le x \le 50 \\ 0 & otherwise \end{cases} \quad \rightarrow E[x] = 25$$

$$f(p) = \begin{cases} 25 & 0 \le p \le 0.05 \\ 0 & otherwise \end{cases} \quad \rightarrow E[p] = 0.025$$

$$f(e_1) = \begin{cases} 25 & 0 \le e_1 \le 0.05 \\ 0 & otherwise \end{cases} \quad \rightarrow E[e_1] = 0.025$$

$$f(e_2) = \begin{cases} 25 & 0 \le e_2 \le 0.05 \\ 0 & otherwise \end{cases} \quad \rightarrow E[e_2] = 0.025$$

Results for the model without replacement of nonconforming items are shown in Table (1). Results for the model with replacement of nonconforming items are shown in Table (2). In addition, results from both models with some sensitivity analysis for various important parameters are compared with [19], when applicable, for the same data used in this example and are also shown in Tables (1 & 2). Based on the sensitivity analysis conducted for various important model parameters, A and h have more effect on the order quantity Q^* than the other parameters. Similarly, C_d and π have less effect on the total expected annual cost than the other parameters.

Figures (1 and 2) show that the expected annual cost is a convex function of the order quantity for the model without replacement of nonconforming items and model with replacement of nonconforming items respectively. The plots of the two annual costs show that the replacement option reduces the expected annual cost.

7. Conclusion

In this paper, EOQ models for stochastic demand are developed for two cases with and without replacement of nonconforming items, which are discoverable during the inspection process; Type I and Type II errors might occur during the inspection process considering 100% inspection policy. The probability of misclassification errors is assumed to be known. The lot fraction nonconforming is assumed to be a random variable following a known distribution. The aim is to find the optimal order quantity and reorder point for each model such that the total cost is minimized. The solution for determining the optimal order quantity and reorder point for both cases is proposed. Numerical examples with some sensitivity analysis for important model parameters are presented for the proposed models.

Possible extensions for future research could be by considering other inspection policy such as sampling inspection policy or considering alternative inventory models such as periodic review. Also, for the nonconforming items, selling them at a discounted price in a maximized-profit model could be considered.

Table (1). Results for model without replacement of nonconforming items.

		EOQ (considering inspection errors)			**EOQ** (not considering inspection errors)		
		Q	r	TAC	Q	r	TAC
C_i	0.5	1288.52	49.69	52,763.41	1256.31	49.69	31,888.98
	1	1288.52	49.69	79,061.90	1256.31	49.69	57,530.01
	1.5	1288.52	49.69	105,360.38	1256.31	49.69	83,171.04
	2	1288.52	49.69	131,658.87	1256.31	49.69	108,812.06
C_d	10	1288.52	49.69	104,702.92	-	-	-
	20	1288.52	49.69	105,031.65	-	-	-
	30	1288.52	49.69	105,360.38	-	-	-
	40	1288.52	49.69	105,689.12	-	-	-
C_n	5	1288.52	49.69	92,539.87	-	-	-
	10	1288.52	49.69	98,950.13	-	-	-
	15	1288.52	49.69	105,360.38	-	-	-
	20	1288.52	49.69	111,770.64	-	-	-
h	1	2880.93	49.86	101,875.97	2808.90	49.86	79,686.62
	3	1663.39	49.76	103,930.49	1621.80	49.76	81,741.14
	5	1288.52	49.69	105,360.38	1256.31	49.69	83,171.04
	8	1018.74	49.61	107,056.84	993.27	49.61	84,867.49
π	5	1289.00	48.77	105,358.09	1256.78	48.77	83,168.74
	10	1288.68	49.39	105,359.62	1256.46	49.39	83,170.27
	20	1288.52	49.69	105,360.38	1256.31	49.69	83,171.04
	30	1288.46	49.80	105,360.64	1256.25	49.80	83,171.29
A	25	743.83	49.82	102,772.52	725.33	49.82	80,583.17
	50	1052.07	49.75	104,236.80	1025.77	49.75	82,047.45
	75	1288.52	49.69	105,360.38	1256.31	49.69	83,171.04
	10	1487.85	49.65	106,307.61	1450.66	49.65	84,118.26

Table (2). Results for model with replacement of nonconforming items.

		EOQ (considering inspection errors)			**EOQ** (not considering inspection errors)		
		Q	r	TAC	Q	r	TAC
C_i	0.5	1224.90	49.69	56,716.71	1224.90	49.69	37,497.96
	1	1224.90	49.69	81,716.71	1224.90	49.69	62,497.96
	1.5	1224.90	49.69	106,716.71	1224.90	49.69	87,497.96
	2	1224.90	49.69	131,716.71	1224.90	49.69	112,497.94
C_r	3	1224.90	49.69	104,216.71	1224.90	49.69	84,997.96
	5	1224.90	49.69	106,716.71	1224.90	49.69	87,497.96
	10	1224.90	49.69	112,966.71	1224.90	49.69	93,747.96
	15	1224.90	49.69	119,216.71	1224.90	49.69	99,997.96
C_a	10	1224.90	49.69	106,091.71	-	-	-
	20	1224.90	49.69	106,404.21	-	-	-
	30	1224.90	49.69	106,716.71	-	-	-
	40	1224.90	49.69	107,029.21	-	-	-
C_n	5	1224.90	49.69	94,529.21	-	-	-
	10	1224.90	49.69	100,622.96	-	-	-
	15	1224.90	49.69	106,716.71	-	-	-
	20	1224.90	49.69	112,810.46	-	-	-
h	1	2738.68	49.86	103,232.29	2738.68	49.86	84,013.54
	3	1581.26	49.76	105,286.81	1581.26	49.76	86,068.06
	5	1224.90	49.69	106,716.71	1224.90	49.69	87,497.96
	8	968.44	49.61	108,413.17	968.44	49.61	89,194.41
π	5	1225.36	48.77	106,714.41	1225.36	48.77	87,495.66
	10	1225.05	49.39	106,715.94	1225.05	49.39	87,497.19
	20	1224.90	49.69	106,716.71	1224.90	49.69	87,497.96
	30	1224.85	49.80	106,716.96	1224.85	49.80	87,498.21
A	25	707.20	49.82	104,128.84	707.20	49.82	84,910.09
	50	1000.13	49.75	105,593.12	1000.13	49.75	86,374.37
	75	1224.90	49.69	106,716.71	1224.90	49.69	87,497.96
	10	1414.39	49.65	107,663.93	1414.39	49.65	88,445.18

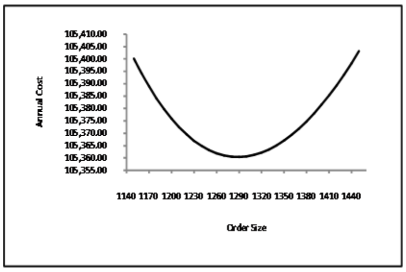

Fig. (1). Expected annual cost versus order quantity for the model without replacement of nonconforming items.

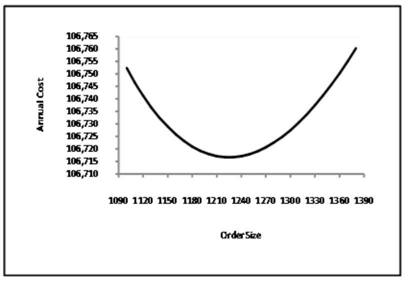

Fig. (2). Expected annual cost versus order quantity for the model with replacement of nonconforming items.

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كمية الطلب الاقتصادية مع الاحتياج العشوائي والجودة غير الكاملة وأخطاء الفحص

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ملخص. في هذه الورقة يتم تطوير النماذج الاقتصادية لكمية الطلب مع الطلب العشوائي للبنود ذات الجودة غير الكاملة وذلك لحالتين؛ مع وبدون استبدال البنود غير المطابقة، أخذاً في الاعتبار نسبة 511% سياسة التفتيش مع افتراض أن تنفيذ عملية تفتيش يتم بصورة غير كاملة. يعتبر الأمر تم وضعه من مورد عند وصول مستوى المخزون إلى نقطة إعادة الطلب. عند تلقي الأمر، سوف يقوم مفتش بتفتيش كل المخزون مع الأخذ في الاعتبار أن المفتش أثناء عملية التفتيش يمكن أن يرتكب أخطاء من النوع I أو النوع II. إن احتمالية حدوث أخطاء "خطأ التصنيف" معروفة. إن الجزء غير المتوافق يفترض أن يكون متغير عشوائي متتبع لتوزيع معروف.

الهدف من هذا البحث هو تحديد كمية الأمر الأمثل للطلب ونقطة إعادة الطلب بحيث يكون التكلفة الإجمالية أقل ما يمكن، مع اقتراح الحل الأمثل لهذا التحديد. يقدم البحث الأمثلة العددية مع تقديم تحليل الحساسية للمعاملات الهامة للنموذج المقترح. مع تقديم التطوير المستقبلي المحتمل للنموذج في القسم الخاص بالاستنتاج.