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Utilization of Dimensional Analysis in the Study of Corona Discharge

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ABSTRACT. In this paper, several attempts to utilize dimensional analysis in the study of corona discharge are presented. Our goal is to determine the dimensionless products pertinent to the study of several corona-related phenomena including (a) corona discharge current, (b) detrimental ozone generation by negative wire-to-plate corona discharge in both dry and humid air, and (c) the small current regime for rod-plane geometry in air. We demonstrate and compare several computational methods to process the dimensional matrix so as to generate the pertinent dimensionless products. Moreover, we contribute a novel explanation (reinforced with ample diverse examples) for the issues of bases, regimes and independent sets of dimensionless products.

Keywords: Dimensional Analysis, Corona Discharge, Ozone Generation, Gauss-Jordan elimination.

1. Introduction

A corona discharge is an electrical discharge, which becomes possible due to the ionization of air surrounding a conductor that is electrically charged [1]–[3]. The corona discharge is another type of strongly non-equilibrium, yet weakly ionized plasma that can be found in nature during electrical storms near sharp edges, points, or thin wires (i.e., near regions of high electric field intensity). A corona discharge can be created with a strong, continuous DC electric field and often requires only one electrode. Corona discharges usually take place at standard atmospheric pressure in contrast to low-temperature (or cold) plasma, which requires a negligibly small absolute pressure or virtually a vacuum. The phenomenon of corona discharge is sometimes called a double-edged sword, since it has both unfavorable and favourable consequences. Despite the several detrimental effects of corona discharges (noise, loss of power, interference with communication signals, and destruction of insulating material) [4]–[7], they currently have several beneficial commercials, scientific and industrial applications [8]–[12].

Investigations of the phenomenon of corona discharge are more likely to be experiment based than to be theory-based. Analysis of or reasoning about this phenomenon is extremely difficult due to the lack of precise explicit knowledge of the physical laws that govern it, and also due to the complexity of the fundamental physical equations that might be proposed to describe it. Therefore, we investigate this phenomenon herein using a method that provides useful (albeit partial) information, while requiring knowledge only of the relevant physical variables and their dimensional representation, namely the method of Dimensional Analysis (DA).

Dimensional Analysis, quite related to the Principle of Similitude, is a tool to present novel analyses of physical systems and phenomena [13]–[20]. Moreover, DA provides physical insight into the dependencies present in physical systems and it reduces the complexity of fundamental physical equations expressing the behaviour of a system to the simplest and most economical form as well [21].

Dimensional Analysis has a long history extending for several centuries, but it was almost one century ago when Buckingham laid the foundation for modern DA [22]. He stated and proved a novel theorem that is now known as the Buckingham Pi Theorem, which can be restated as follows: "If there exists a dimensionally homogeneous equation

$$f(A_1, A_2, A_3, \ldots, A_n) = 0$$

among *n* physical quantities, which involve *k* physical dimensions ($k < n$), then there also exists a relation

$$\Phi\ (\pi_1, \pi_2, \pi_3, \ldots, \pi_m)\ =\ 0$$

among *m* independent dimensionless products comprised of powers of the pertinent variables A_i's, where $(n - k) \leq m < n$." In fact, *m* is equal to $(n - r)$, where *r* is the *rank* of the *k* by *n dimensional matrix* formed by the dimensional exponents of the A_i's [13].

In this study, DA is applied to determine dimensionless products among physical quantities pertaining to corona discharge. The ultimate DA result is a general expression that describes several aspects of corona discharge including the impact of the sharp edge area of lightning rod tips to corona discharge current, the effect of humidity on ozone generation and the influence of the tip radius and gap length on the discharge current.

The remainder of this paper is structured as follows. Section 2 is divided into three subsections devoted to brief introductions to the three problems of corona discharge considered herein. Section 3 offers a quick introduction to the DA method, followed by DA application to the afore-mentioned problems. The DA results are used in qualitative reasoning about the pertinent problems. As a bonus, Section 3 also includes a novel and informative discussion (with ample detailed examples) of the concepts of bases and regimes as well as those of independence and completeness for dimensionless products. In Section 4, discussions and visualizations are presented and Section 5 concludes the paper.

2. Some Corona Discharge Problems

Corona discharge has many related phenomena or problems such as those of corona discharge current magnitude [23], detrimental ozone generation by negative wire-to-plate corona discharge in both dry and humid air [24], [25], and the small current regime for rod-plane geometry in the air [26]. These problems are described in the following subsections.

2.1 Corona Discharge Current Magnitude

The performance of several lightning rod tips such as pointed, blunt (unsharpened), conical and flat rod tips for a lightning air terminal has been investigated by Sidik et al. [23]. To obtain the behaviour of several lightning rod tips, laboratory tests were performed. Employing dimensional analysis, we can scientifically confirm the intuitionistic conjecture that increasing surface sharpness can increase the corona discharge current.

2.2 Detrimental Ozone Generation

Corona discharges of indoor electrostatic devices, such as laser printers, photocopiers, and electrostatic precipitators have a notorious impact of contributing to the generation of the detrimental ozone, an air pollutant that triggers a variety of health problems. In Ref. [24], the study of dimensional analysis for the negative wire to plate corona discharge has been targeted to exploring the effect of humidity on the ozone generation.

To obtain a basic relationship of the system, dimensional analysis on the dry air negative wire to plate corona discharge has been utilized. Furthermore, exploration of the impact of humidity on ozone generation led to a more comprehensive relationship that is potentially applicable for real operation conditions of many indoor corona devices.

2.3 The Small Current Regime for Rod-Plane Geometry in Air

Jones et al. [26] explain the influence of tip radius and gap length on the inception regime for the simple electrode geometry of a hemi-spherically tipped cylindrical rod opposite a plane electrode. There are four steps towards obtaining the clarification, namely, implementation of dimensional analysis, experimental measurement, comparison of the predictions of dimensional analysis with measured data and attempts at physical interpretation and qualitative reasoning.

3. Dimensional Analysis (DA)

Dimensional analysis is a beneficial procedure of particular importance in experiment-based areas of engineering. Dimensional analysis can be employed to define the relationship among physical quantities using only their physical dimensions and can provide the general forms of equations that describe natural phenomena [27] [28]. If the factors involved in a physical situation can be determined (without any missing or extraneous ones), dimensional analysis can be used to correctly reveal the relationship between them.

3.1 Dimensional Analysis on Corona Discharge Current

The corona discharge current I_{cd} depends on a number of variables that have some observable influence on the phenomenon of corona discharge [23]. The variables involved are permittivity ε, time T, the area of the tip that produces the corona A, distance d between electrodes, and the input voltage V. The variables can be represented in terms of the four most fundamental dimensions (taken arbitrarily here as mass $[M]$, length $[L]$, time $[T]$ and electric charge $[Q]$) as shown in Table (1). This $MLTQ$ dimensional basis might be replaced by the $MLTI$ dimensional basis or the $LTI\phi$ dimensional basis, where $[I]$ is the electric current dimension, while $[\phi]$ is that of the electric potential or voltage.

Table (1): Variables involved in the corona discharge current problem.

Variable	Symbol	Dimensions
Discharge Current	I_{cd}	$T^{-1}Q^{1}$
Permittivity	ε	$M^{-1}L^{-3}T^{2}Q^{2}$
Time	T	T^{1}
The area of the tip that produces the corona	A	L^{2}
Distance	d	L^{1}
voltage	V	$M^{1}L^{2}T^{-2}Q^{-1}$

According to Buckingham theorem [22], each dimensionless product of the six variables in Table (1) will be of the form

$$\pi = k\, A^{e}\, T^{f}\, \varepsilon^{g}\, {I_{cd}}^{h}\, d^{i}\, V^{j} \qquad (1)$$

where k is a dimensionless constant and $e, f, g, h, i,$ and j are exponents yet to be inter-related partially. If we denote by $[x]$ the dimension of the quantity x, we note that $[\pi] = [k] = 1$, where

$$[\pi] = (L^2)^e (T^1)^f (M^{-1}L^{-3}T^2Q^2)^g (T^{-1}Q^1)^h (L^1)^i (M^1L^2T^{-2}Q^{-1})^j \quad (2)$$

or equivalently

$$M^0L^0T^0Q^0 = M^{-g+j}L^{2e-3g+i+2j}T^{f+2g-h-2j}Q^{2g+h-j} \quad (3)$$

The product π is dimensionless if

$$-g + j = 0 \quad (4a)$$

$$2e - 3g + i + 2j = 0 \quad (4b)$$

$$f + 2g - h - 2j = 0 \quad (4c)$$

$$2g + h - j = 0 \quad (4d)$$

In equations (4) we have six unknown exponents $e, f, g, h, i,$ and j but only four linear equations. We will see shortly that these four equations are linearly independent, which means that the dimensional matrix comprising them has full rank (r) of 4. Therefore, based on the Buckingham π theorem there are two independent dimensionless products.

Equations (4) are rewritten as a single matrix equation

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 1 \\ 2 & 0 & -3 & 0 & 1 & 2 \\ 0 & 1 & 2 & -1 & 0 & -2 \\ 0 & 0 & 2 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \\ i \\ j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

The dimensional-matrix form in (5) is rewritten as shown in Table (2) [1]. Using Gauss-Jordan reduction by implementing permissible elementary row operations [13], we are in a position to assert that our dimensional matrix has a full rank (The Gauss-Jordan procedure terminates without creating an all-0 row), and consequently to determine the pertinent two dimensionless products π_1 and π_2.

The final result of Table (2) expresses each of the four indices $e, f, g,$ and h (called basis indices [13]) in terms of the two remaining indices i and j (called regime indices [13] as

$$e = -\frac{1}{2}i + \frac{1}{2}j \tag{6a}$$

$$f = -j \tag{6b}$$

$$g = j \tag{6c}$$

$$h = -j \tag{6d}$$

We substitute Eq. (6) into Eq. (1) to obtain

$$\pi = k\left(\frac{d}{A^{0.5}}\right)^{i}\left(\frac{V\varepsilon A^{0.5}}{TI_{cd}}\right)^{j} \tag{7}$$

So, finally we get two independent dimensionless products, namely

$$\pi_1 = \left(\frac{d}{A^{0.5}}\right) \tag{8a}$$

$$\pi_2 = \left(\frac{V\varepsilon A^{0.5}}{TI_{cd}}\right) \tag{8b}$$

Table (2): Application of Gauss-Jordan Elimination to the matrix equation (5)

	e	f	g	h	i	j	
E_1	0	0	−1	0	0	1	0
E_2	2	0	−3	0	1	2	0
E_3	0	1	2	−1	0	−2	0
E_4	0	0	2	1	0	−1	0
E_1	0	0	−1	0	0	1	0
E_2	2	0	−3	0	1	2	0
E_3+E_4	0	1	4	0	0	−3	0
E_4	0	0	2	1	0	−1	0
E_1	0	0	−1	0	0	1	0
E_2	2	0	−3	0	1	2	0

Table 2 Continued

	e	f	g	h	i	j	
$\frac{E_3}{4}$	0	$\frac{1}{4}$	1	0	0	$-\frac{3}{4}$	0
E_4	0	0	2	1	0	-1	0
E_1+E_3	0	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	0
E_2+3E_3	2	$\frac{3}{4}$	0	0	1	$-\frac{1}{4}$	0
E_3	0	$\frac{1}{4}$	1	0	0	$-\frac{3}{4}$	0
E_4-2E_3	0	$-\frac{1}{2}$	0	1	0	$\frac{1}{2}$	0
E_1	0	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	0
$\frac{4}{3}E_2$	$\frac{8}{3}$	1	0	0	$\frac{4}{3}$	$-\frac{1}{3}$	0
E_3	0	$\frac{1}{4}$	1	0	0	$-\frac{3}{4}$	0
E_4	0	$-\frac{1}{2}$	0	1	0	$\frac{1}{2}$	0
$E_1-\frac{1}{4}E_2$	$-\frac{2}{3}$	0	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
E_2	$\frac{8}{3}$	1	0	0	$\frac{4}{3}$	$-\frac{1}{3}$	0
$E_3-\frac{1}{4}E_2$	$-\frac{2}{3}$	0	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$E_4+\frac{1}{2}E_2$	$\frac{4}{3}$	0	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0
$-\frac{3}{2}E_1$	1	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
E_2	$\frac{8}{3}$	1	0	0	$\frac{4}{3}$	$-\frac{1}{3}$	0
E_3	$-\frac{2}{3}$	0	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0
E_4	$\frac{4}{3}$	0	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0
E_1	1	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0

Table 2 Continued

	e	f	g	h	i	j	
$E_2 - \frac{8}{3}E_1$	0	1	0	0	0	1	0
$E_3 + \frac{2}{3}E_1$	0	0	1	0	0	−1	0
$E_4 - \frac{4}{3}E_1$	0	0	0	1	0	1	0

The other method to determine π_1 and π_2 is using explicit matrix inversion. Partitioning the dimensional matrix in Eq. (5), we can define the square matrix $\boldsymbol{A}$ (comprising the first four columns, the four basis columns) and the matrix $\boldsymbol{B}$ (comprising the remaining two columns, the two regime columns) as follows

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 2 & 0 & -3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 0 & -2 \\ 0 & -1 \end{bmatrix}$$

and then we get the inverse $\boldsymbol{A^{-1}}$ of the matrix $\boldsymbol{A}$, which is guaranteed to exist under the prior full-rank assumption for this matrix

$$\boldsymbol{A^{-1}} = \begin{bmatrix} -1.5 & 0.5 & 0 & 0 \\ 4 & 0 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

The next step is to calculate a new matrix $\boldsymbol{C}$, defined as follows

$$\boldsymbol{C} = -(\boldsymbol{A^{-1}B})^T = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0.5 & -1 & 1 & -1 \end{bmatrix}$$

Table (3): Placing matrix $\boldsymbol{C}$ beside an appropriate unit matrix

	A	T	ε	I_{cd}	d	V
π_1	-0.5	0	0	0	1	0
π_2	0.5	-1	1	-1	0	1

Table (3) places matrix $\boldsymbol{C}$ besides an appropriate unit matrix (of the same number of rows). Each row in the overall matrix designates a specific dimensionless product. The entries in each row are the indices of the respective variables needed to construct the designated dimensionless product. The resulting values for π_1 and π_2 could be seen to be exactly equal to those in Eqs. (8).

The last method to find π_1 and π_2 is one using Fundamental Solutions. Again, this method assumes prior knowledge of the rank of the dimensional matrix. In our current problem, this rank is assumed by the method to be 4, and hence there are 2 fundamental solutions, called fundamental modes or regimes. For the first fundamental solution, we assume $i = 1 \; and \; j = 0,$ so that Eq. (5) takes the form $\boldsymbol{A}\,\boldsymbol{x_1} = -\,\boldsymbol{b_1}$, where matrix $\boldsymbol{A}$ is as given before and vector $\boldsymbol{b_1}$ is the first column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_1}$ is a vector representing the first fundamental-mode value of the basis indices $[e \; f \; g \; h]^{\mathrm{T}}$ and is given by

$$\boldsymbol{x_1} = -\boldsymbol{A^{-1}}\boldsymbol{b_1} = \begin{bmatrix} -0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 1, j = 0$), we get the set of indices for π_1, which turns out to be equal to that in Eq. (8a).

For the second Fundamental Solution, we assume $i = 0 \; and \; j = 1,$ and then Eq. (5) takes the form $\boldsymbol{A}\,\boldsymbol{x_2} = -\,\boldsymbol{b_2}$ where matrix $\boldsymbol{A}$ is as before while vector $\boldsymbol{b_2}$ is the second column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_2}$ is a vector representing the second fundamental-mode value of the basis indices $[e \; f \; g \; h]^{\mathrm{T}}$, and is given by

$$\boldsymbol{x_2} = -\boldsymbol{A^{-1}b_2} = \begin{bmatrix} -0.5 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 0, j = 1$), we get the set of indices for π_2 , which turns out to be equal to that in Eq. (8b).

Now, we study the possible bases, regimes and independent dimensionless products for the current problem. There are at most $\binom{6}{4} = \binom{6}{2} = 15$ pairs of dimensionless products that form independent regimens as shown in Table (4). We now digress a little bit to explain how and when the pairs of independent products in Table (4) are obtained. For this purpose, we reproduce below Eq. (7) with the dimensionless constant k in it set to 1. So that Eq. (7) becomes

$$\pi = \left(\frac{d}{A^{0.5}}\right)^i \left(\frac{V\varepsilon A^{0.5}}{TI_{cd}}\right)^j = (\pi_{a1})^i (\pi_{b1})^j \qquad \text{(7a)}$$

Table (4): Pairs of dimensionless products that form independent regimes

Case i	Set of basis (input) variable	Set of isolated (output) variables	π_{ai}	π_{bi}
1	$\{A, T, \varepsilon, I_{cd}\}$	$\{d, V\}$	$\pi_{a1} = \frac{d}{A^{0.5}}$	$\pi_{b1} = \frac{V\varepsilon A^{0.5}}{TI_{cd}}$
2	$\{A, T, \varepsilon, d\}$	$\{I_{cd}, V\}$	−	−
3	$\{A, T, I_{cd}, d\}$	$\{\varepsilon, V\}$	−	−
4	$\{A, \varepsilon, I_{cd}, d\}$	$\{T, V\}$	−	−
5	$\{T, \varepsilon, I_{cd}, d\}$	$\{A, V\}$	$\pi_{a5} = {\pi_{a1}}^{-2}$	$\pi_{b5} = \pi_{a1}\pi_{b2}$
6	$\{A, T, \varepsilon, V\}$	$\{I_{cd}, d\}$	$\pi_{a6} = {\pi_{b2}}^{-1}$	$\pi_{b6} = \pi_{a1}$
7	$\{A, T, I_{cd}, V\}$	$\{\varepsilon, d\}$	$\pi_{a7} = \pi_{b2}$	$\pi_{b7} = \pi_{a1}$
8	$\{A, \varepsilon, I_{cd}, V\}$	$\{T, d\}$	$\pi_{a8} = \pi_{a6}$	$\pi_{b8} = \pi_{a1}$
9	$\{T, \varepsilon, I_{cd}, V\}$	$\{A, d\}$	$\pi_{a9} = {\pi_{b2}}^2$	$\pi_{b9} = \pi_{b5}$
10	$\{A, T, d, V\}$	$\{\varepsilon, I_{cd}\}$	−	−
11	$\{A, \varepsilon, d, V\}$	$\{T, I_{cd}\}$	−	−

12	$\{T, \varepsilon, d, V\}$	$\{A, I_{cd}\}$	$\pi_{a12} = \pi_{a5}$	$\pi_{b12} = {\pi_{b5}}^{-1}$
13	$\{A, I_{cd}, d, V\}$	$\{T, \varepsilon\}$	$-$	$-$
14	$\{T, I_{cd}, d, V\}$	$\{A, \varepsilon\}$	$\pi_{a14} = \pi_{a5}$	$\pi_{b14} = \pi_{b5}$
15	$\{\varepsilon, I_{cd}, d, V\}$	$\{A, T\}$	$\pi_{a15} = \pi_{a5}$	$\pi_{b15} = \pi_{b12}$

To obtain the pair of products in line 1 of Table (4), we simply set (i, j) to the two independent values (1,0) and (0,1), thereby obtaining the pair π_{a1} and π_{b1}. To obtain the pair of products π_{a2} and π_{b2} anticipated to appear in line 2 of Table (4), we note that π_{a2} is a regime for I_{cd}, associated with π_{b2} being a regime for V. The exponent of I_{cd} should be 1 in π_{a2} and 0 in π_{b2}, while the exponent of V should be 0 in π_{a2} and 1 in π_{b2}. These are conditions that are impossible to satisfy simultaneously. Therefore, the I_{cd} and V variables cannot be made two regime variables simultaneously, and our attempt at obtaining the purported pair of products π_{a2} and π_{b2} in line 2 of Table (4) is aborted. In fact, the selection of I_{cd} and V as regime variables is equivalent to a column interchange in the original dimensional matrix that would produce a non-invertible A matrix, despite the fact that the original dimensional matrix has a full rank (and hence, the Gauss-Jordan procedure would react by enforcing a reverse column interchange, thereby aborting the situation of having the I_{cd} and V variables as regime variables simultaneously). Next, we consider how to obtain the pair of products π_{a5} and π_{b5} in line 5 of Table (4), We note that π_{a5} is a regime for A, associated with π_{b5} being a regime for V. The exponent of A should be 1 in π_{a5} and 0 in π_{b5}, while the exponent of V should be 0 in π_{a5} and 1 in π_{b5}: conditions that are easily satisfied simultaneously through the choices made in line 5 of the table. The rest of the table is handled as we did for line 2 or line 5.

3.2 Dimensional Analysis of Detrimental Ozone Generation

In dry air discharge, a dimensional system was constructed by involving seven physical parameters namely the ozone generation rate per unit length of wire in dry air r^0, the wire radius a, the applied voltage V, the inter-electrode gap d, the excess voltage V_e, the permittivity for the inter-electrode drift region ε, and the ion mobility μ [24]. The four fundamental dimensions $MLTI$ are used in Table (5) to represent these seven variables.

Table (5): Variables involved in the problem of detrimental ozone generation

Variable	Symbol	Dimensions
The ozone generation rate per unit length of wire in dry air	r^0	$M^1L^{-1}T^{-1}$
The wire radius	a	L^1
The applied voltage	V	$M^1L^2T^{-3}A^{-1}$
The inter-electrode gap	d	L^1
excess voltage	V_e	$M^1L^2T^{-3}A^{-1}$
The permittivity for the inter-electrode drift region	ε	$M^{-1}L^{-3}T^4A^2$
The ion mobility	μ	$M^{-1}T^2A^1$

According to Buckingham theorem [22], each dimensionless product of the seven variables in Table (5) will be of the form

$$\pi = k\mu^e\varepsilon^f {V_e}^g d^h {r^0}^i a^j V^l \qquad (9)$$

where k is a dimensionless constant and $e, f, g, h, i, j,$ and l are exponents yet to be determined partially. If we denote by $[x]$ the dimension of the quantity x, we note that $[\pi] = [k] = 1$, where

$$\pi = (M^{-1}T^2A^1)^e(M^{-1}L^{-3}T^4A^2)^f(M^1L^2T^{-3}A^{-1})^g$$

$$(L^1)^h(M^1L^{-1}T^{-1})^i(L^1)^j(M^1L^2T^{-3}A^{-1})^l \qquad (10)$$

or equivalently

$$M^0L^0T^0Q^0 = M^{-e-f-g+i+l}L^{-3f+2g+h-i+j+2l}T^{2e+4f-3g-i-3l}A^{e+2f-g-l} \qquad (11)$$

The product of π is dimensionless if

$$-e - f - g + i + l = 0 \tag{12a}$$

$$-3f + 2g + h - i + j + 2l = 0 \tag{12b}$$

$$2e + 4f - 3g - i - 3l = 0 \tag{12c}$$

$$e + 2f - g - l = 0 \tag{12d}$$

In equations (12) we have seven unknown exponents e, f, g, h, i, j and l but only 4 equations. We will see shortly that these four equations are linearly independent, which means that the dimensional matrix comprising them has a full rank (r) of 4. Therefore, based on the Buckingham π theorem, there are three independent dimensionless products.

Equation (12) is rewritten as a single matrix equation

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 1 & 0 & 1 \\ 0 & -3 & 2 & 1 & -1 & 1 & 2 \\ 2 & 4 & -3 & 0 & -1 & 0 & -3 \\ 1 & 2 & -1 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \\ i \\ j \\ l \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

The dimensional-matrix form in (13) is rewritten as shown in Table (6) [1]. Using Gauss-Jordan reduction by implementing permissible elementary row operations [13], we are in a position to assert that our dimensional matrix has full rank (The Gauss-Jordan procedure terminates without creating an all-0 row), and consequently to determine the pertinent three dimensionless products π_1, π_2 and π_3.

The final result of Table (6) expresses each of the four indices $e, f, g,$ and h (called basis indices [13]) in terms of the two remaining indices i, j and l (called regime indices [13] as

$$e = i \tag{14a}$$

$$f = -i \tag{14b}$$

$$g = -i - l \tag{14c}$$

$$h = -j \tag{14d}$$

We substitute Eq. (14) into Eq. (9), to obtain

$$\pi = k\left(\frac{r^0\mu}{\varepsilon V_e}\right)^i \left(\frac{a}{d}\right)^j \left(\frac{V}{V_e}\right)^l \tag{15}$$

So, finally we get three independent dimensionless products, namely

$$\pi_1 = \left(\frac{r^0\mu}{\varepsilon V_e}\right) \tag{16a}$$

$$\pi_2 = \left(\frac{a}{d}\right) \tag{16b}$$

$$\pi_3 = \left(\frac{V}{V_e}\right) \tag{16c}$$

The other method to determine π_1, π_2 and π_3 is using explicit matrix inversion. Partitioning the dimensional matrix in Eq. (13), we can define the square matrix $\boldsymbol{A}$ (comprising the first four columns, the four basis columns) and the matrix $\boldsymbol{B}$ (comprising the remaining three columns, the three regime columns) as follows

$$\boldsymbol{A} = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 0 & -3 & 2 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & -1 & 0 \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ -1 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

and then we get the inverse A^{-1} of the matrix A, which is guaranteed to exist under the prior full-rank assumption for this matrix

$$\boldsymbol{A^{-1}} = \begin{bmatrix} -2 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 3 & 1 & 2 & -1 \end{bmatrix}$$

The next step is to calculate a new matrix C, defined as follows

$$\boldsymbol{C} = -(\boldsymbol{A}^{-1}\boldsymbol{B})^T = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Table (6): Application of Gauss-Jordan Elimination to the matrix equation (13)

	e	f	g	h	i	j	l	
E_1	-1	-1	1	0	1	0	1	0
E_2	0	-3	2	1	-1	1	2	0
E_3	2	4	-3	0	-1	0	-3	0
E_4	1	2	-1	0	0	0	-1	0
$-E_1$	1	1	-1	0	-1	0	-1	0
E_2	0	-3	2	1	-1	1	2	0
E_3	2	4	-3	0	-1	0	-3	0
E_4	1	2	-1	0	0	0	-1	0
E_1	1	1	-1	0	-1	0	-1	0
E_2	0	-3	2	1	-1	1	2	0
$E_3 - 2E_1$	0	2	-1	0	1	0	-1	0
$E_4 - E_1$	0	1	0	0	1	0	0	0
E_1	1	1	-1	0	-1	0	-1	0
$-\frac{E_2}{3}$	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
E_3	0	2	-1	0	1	0	-1	0
E_4	0	1	0	0	1	0	0	0

$E_1 - E_2$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0
E_2	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$E_3 - 2E_2$	0	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$E_4 - E_2$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0
E_1	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0
E_2	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
Table (6) Continued								
	e	f	g	h	i	j	l	
$3E_3$	0	0	1	-2	1	2	1	0
E_4	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	0
$E_1 + \frac{E_3}{3}$	1	0	0	1	-1	1	0	0
$E_2 + \frac{2E_3}{3}$	0	1	0	1	1	1	0	0
E_3	0	0	1	2	1	2	1	0
$E_4 - \frac{2E_3}{3}$	0	0	0	-1	0	-1	0	0
E_1	1	0	0	1	-1	1	0	0
E_2	0	1	0	1	1	1	0	0
E_3	0	0	1	2	1	2	1	0
$-E_4$	0	0	0	1	0	1	0	0
$E_1 - E_4$	1	0	0	0	-1	0	0	0

$E_2 - E_4$	0	1	0	0	1	0	0	0
$E_3 - 2E_4$	0	0	1	0	1	0	1	0
E_4	0	0	0	1	0	1	0	0

Table (7): Placing matrix $\boldsymbol{C}$ beside an appropriate unit matrix

	μ	ε	V_e	d	r^0	a	V
π_1	1	-1	-1	0	1	0	0
π_2	0	0	0	-1	0	1	0
π_3	0	0	-1	0	0	0	1

Table (7) places matrix $\boldsymbol{C}$ besides an appropriate unit matrix (of the same number of rows). Each row in the overall matrix designates a specific dimensionless product. The entries in each row are the indices of the respective variables needed to construct the designated dimensionless product. The resulting values for π_1, π_2 and π_3 could be seen to be exactly equal to those in Eqs. (16).

The last method to find π_1, π_2 and π_3 is using Fundamental Solution. Again, this method assumes prior knowledge of the rank of the dimensional matrix. In our current problem, this rank is assumed by the method to be 4, and hence there are 3 fundamental solutions, called fundamental modes or regimes. For the first fundamental solution, we assume $i = 1\ j = 0\ and\ l = 0$ so that Eq. (13) takes the form $\boldsymbol{A}\,\boldsymbol{x_1} = -\,\boldsymbol{b_1}$, where matrix $\boldsymbol{A}$ is as given before and vector $\boldsymbol{b_1}$ is the first column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_1}$ is a vector representing the first fundamental-mode value of the basis indices $[e\ f\ \ g\ h]^{\mathrm{T}}$ and is given by

$$\boldsymbol{x_1} = -\boldsymbol{A^{-1}}\boldsymbol{b_1} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 1\ j = 0\ and\ l = 0$), we get the set of indices for π_1, which turns out to be equal to that in Eq. (16a).

For the second Fundamental Solution, we assume $i = 0\ j = 1\ and\ l = 0$, and then Eq. (13) takes the form $\boldsymbol{A}\,\boldsymbol{x_2} = -\,\boldsymbol{b_2}$ where matrix $\boldsymbol{A}$ is as before while vector $\boldsymbol{b_2}$ is the second column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_2}$ is a vector representing the second fundamental-mode value of the basis indices $[e\ f\ g\ h]^{T}$, and is given by

$$\boldsymbol{x_2} = -\boldsymbol{A^{-1}b_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 0\ j = 1\ and\ l = 0$), we get the set of indices for π_2 , which turns out to be equal to that in eq. (16b).

For the third fundamental solution, we assume $i = 0\ j = 0\ and\ l = 1$, and then Eq. (13) takes the form $\boldsymbol{A}\,\boldsymbol{x_3} = -\,\boldsymbol{b_3}$ where matrix $\boldsymbol{A}$ is as before while vector $\boldsymbol{b_3}$ is the third column in matrix $\boldsymbol{B}$ above. The vector $\boldsymbol{x_3}$ is a vector representing the second fundamental-mode value of the basis indices $[e\ f\ g\ h]^{T}$, and is given by

$$\boldsymbol{x_3} = -\boldsymbol{A^{-1}b_2} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 0\ j = 0\ and\ l = 1$), we get the set of indices for π_3 , which turns out to be equal to that in Eq. (16b).

3.3 Dimensional Analysis on Rod-Plane Geometry for Currents Near Inception

In [26], the system involves seven physical parameters namely the potential between electrodes V, the overall corona current I, the average mobility value μ, the permittivity ε, the characteristic separation d, and two indicators of the shapes of two electrodes that given by the respective characteristic lengths a and A. The four fundamental dimensions $MLQT$ are used in Table (8) to represent these seven variables. Three of these variables (d, A and a) are lengths, and only the first among them (the characteristic separation d) is deemed of significant importance. As a first cut at the problem, the shape indicators A and a are excluded from further consideration. Their inclusion would unnecessarily complicate the analysis, and we would be unable to see the wood for the trees. With the exclusion of the shape indicators, we will be in a position to get a general understanding of the situation, without worrying (for the moment) about minute details.

Table (8): Variables involved in the problem of subsection 3.3

Variable	Symbol	Dimensions
The potential between electrodes	V	$M^1L^2Q^{-1}T^{-2}$
The overall corona current	I	Q^1T^{-1}
An average mobility value	μ	$M^{-1}Q^1T^1$
The permittivity	ε	$M^{-1}L^{-3}Q^2T^2$
the characteristic separation	d	L^1
the shape indicator of the first electrodes	A	L^1
the second shape indicator of the second electrode	a	L^1

Based on Table (8) and according to Buckingham theorem [22], each dimensionless product of Table (8) will be of the form

$$\pi = kV^e d^f \varepsilon^g \mu^h I^i \tag{17}$$

where k is a dimensionless constant and $e, f, g, h, i,$ and j are exponents yet to be determined partially. If we denote by $[x]$ the dimension of the quantity x, we note that$[\pi] = [k] = 1$, where

$$\pi = (M^1L^2Q^{-1}T^{-2})^e(L^1)^f(M^{-1}L^{-3}Q^2T^2)^g(M^{-1}Q^1T^1)^h(Q^1T^{-1})^i \tag{18}$$

or equivalently

$$M^0L^0Q^0T^0 = M^{e-g-h}L^{2e+f-3g}T^{-2e+2g+h}Q^{-e+2g+h+i} \tag{19}$$

The product of π is dimensionless if

$$e \quad - \quad g - h \quad = 0 \tag{20a}$$

$$2e + f - 3g \quad = 0 \tag{20b}$$

$$-e \quad + 2g + h \ + i = 0 \tag{20c}$$

$$-2e \quad + 2g + h - i = 0 \tag{20d}$$

In equations (20) we have five unknowns exponents $e, f, g, h,$ and i but only 4 equations. We will see shortly that these four equations are linearly independent, which means that the dimensional matrix comprising them has a full rank (r) of 4. Therefore, based on the Buckingham π theorem there is only a single independent dimensionless product.

Equation (20) are rewritten as a single matrix equation

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 \\ 2 & 1 & -3 & 0 & 0 \\ -1 & 0 & 2 & 1 & 1 \\ -2 & 0 & 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

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The dimensional-matrix form in (21) is rewritten as shown in Table (9). Using Gauss-Jordan reduction by implementing permissible elementary row operations [13], we are in a position to assert that our dimensional matrix has full rank (The Gauss-Jordan procedure terminates without creating an all-0 row), and consequently to determine the pertinent one dimensionless products π_1.

The final result of Table (9) expresses each of the four indices $e, f, g,$ and h (called basis indices [13]) in terms of the one remaining index i (called regime index [13] as

$$e = -2i \tag{22a}$$

$$f = i \tag{22b}$$

$$g = -i \tag{22c}$$

$$h = -i \tag{22d}$$

We substitute Eq. (22) into Eq. (17), then to obtain

$$\pi = k\left(\frac{Id}{V^2\varepsilon\mu}\right)^i \tag{23}$$

Table (9): Application of Gauss-Jordan Elimination to the matrix equation (21)

	e	f	g	h	i	
E_1	1	0	−1	−1	0	0
E_2	2	1	−3	0	0	0
E_3	−1	0	2	1	1	0
E_4	−2	0	2	1	−1	0
E_1	1	0	−1	−1	0	0
$E_2 - E_1$	0	1	−1	2	0	0
$E_3 + E_1$	0	0	1	0	1	0
$E_4 + 2E_1$	0	0	0	−1	−1	0
$E_1 + E_3$	1	0	0	−1	1	0

$E_2 - E_3$	0	1	0	2	1	0
E_3	0	0	1	0	1	0
E_4	0	0	0	-1	-1	0
E_1	1	0	0	-1	1	0
E_2	0	1	0	2	1	0
E_3	0	0	1	0	1	0
$-E_4$	0	0	0	1	1	0
E_1	1	0	0	0	2	0
E_2	0	1	0	0	-1	0
E_3	0	0	1	0	1	0
E_4	0	0	0	1	1	0

So, finally we get one independent dimensionless product, namely

$$\pi_1 = \left(\frac{Id}{V^2 \varepsilon \mu}\right) \tag{24}$$

The other method to determine π_1 is using explicit matrix inversion. Partitioning the dimensional matrix in Eq. (21), we can define the square matrix $\boldsymbol{A}$ (comprising the first four columns, the four basis columns) and the matrix $\boldsymbol{B}$ (comprising the remaining column, the one regime column) as follows

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 2 & 1 & -3 & 0 \\ -1 & 0 & 2 & 1 \\ -2 & 0 & 2 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

and then we get the inverse $\boldsymbol{A}^{-1}$ of the matrix $\boldsymbol{A}$, which is guaranteed to exist under the prior full-rank assumption for this matrix

$$\boldsymbol{A^{-1}} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 3 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ -2 & 0 & 0 & -1 \end{bmatrix}$$

The next step is to calculate a new matrix C, defined as follows

$$\boldsymbol{C} = -(\boldsymbol{A^{-1}B})^T = [-2 \quad 1 \quad -1 \quad -1]$$

Table (10): Placing matrix *C* beside an appropriate unit matrix

	V	d	ε	μ	I
π_1	-2	1	-1	-1	1

Table (10) places matrix $\boldsymbol{C}$ besides an appropriate unit matrix (of the same number of rows). Each row in the overall matrix designates a specific dimensionless product. The entries in each row are the indices of the respective variables needed to construct the designated dimensionless product. The resulting values for π_1 could be seen to be exactly equal to those in Eqs. (23).

The last method to find π_1 is one using Fundamental Solutions. Again, this method assumes prior knowledge of the rank of the dimensional matrix. In our current problem, this rank is assumed by the method to be 4, and hence there are 1 fundamental solution, called fundamental modes or regimes. For the fundamental solution, we assume $i = 1$ so that Eq. (21) takes the form $\boldsymbol{Ax} = -\boldsymbol{b}$, where matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ are given before and $\boldsymbol{x}$ is a vector representing the first fundamental-mode value of the basis indices $[e \; f \; g \; h]^{\mathrm{T}}$ and is given by

$$\boldsymbol{x} = -\boldsymbol{A}^{-1}\boldsymbol{b} = \begin{bmatrix} -2 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

When we augment this value of basis indices by the assumed regime indices ($i = 1$), we get the set of indices for π_1, which turns out to be equal to that in Eq. (23).

Now, we study the possible bases, regimes and independent dimensionless products for the current problem. There are at most $\binom{5}{1} = \binom{5}{4} = 5$ pairs of dimensionless products that form independent regimens as shown in Table (11). We now digress a little bit to explain how and when the pairs of independent products in Table (11) are obtained. For this purpose, we reproduce below Eq. (23) with the dimensionless constant k in it set to 1. So that Eq. (23) becomes

$$\pi = \left(\frac{Id}{V^2 \varepsilon \mu}\right)^i = (\pi_{a1})^i \tag{23a}$$

Table (11): Pairs of dimensionless products that form independent regimes

Case i	Set of basis (input) variable	Set of isolated (output) variables	π_{ai}
1	$\{V, d, \varepsilon, \mu\}$	$\{I\}$	$\pi_{a1} = \dfrac{Id}{V^2 \varepsilon \mu}$
2	$\{V, d, \varepsilon, I\}$	$\{\mu\}$	$\pi_{a2} = {\pi_{a1}}^{-1}$
3	$\{V, d, \mu, I\}$	$\{\varepsilon\}$	$\pi_{a3} = \pi_{a2}$
4	$\{V, \varepsilon, \mu, I\}$	$\{d\}$	$\pi_{a4} = \pi_{a1}$
5	$\{d, \varepsilon, \mu, I\}$	$\{V\}$	$\pi_{a5} = {\pi_{a1}}^{-0.5}$

To obtain the pair of products in line 1 of Table (11), we simply set $i = 1$, thereby obtaining the π_{a1}. To obtain the pair of products π_{a2} in line 2 of Table (11), we note that π_{a2} is a regime for μ.

4. Discussion and Visualization

In this section, we visualize the effects of particular parameters for the system by simulating and analyzing corona discharge phenomena using matlab Simulink for both the corona discharge current and the detrimental ozone generation effects. First, we consider the corona discharge current, for which the general DA model suggested by Eqs. (8) leads to relating the dimensionless constants therein via an arbitrary function. We choose to write

$$\pi_2^{-1} = f(\pi_1^{-1}) \tag{24}$$

$$\left(\frac{V\varepsilon A^{0.5}}{TI_{cd}}\right)^{-1} = f\left(\left(\frac{d}{A^{0.5}}\right)^{-1}\right) \tag{25}$$

and hence the corona discharge current,, might be expressed as follows

$$I_{cd} = \frac{V\ \varepsilon\ A^{0.5}}{T}\ f\left(\frac{A^{0.5}}{d}\right) = \frac{V\ \varepsilon\ d}{T}\ g\left(\frac{A^{0.5}}{d}\right) \tag{26}$$

Sidik er al. [23] used curve-fitting of results obtained through experimental measurements to obtain an empirical relation between the area of the sharp edge and the I_{cd} magnitude, namely

$$I_{cd} = 84.579\ A^{0.2508} \tag{27}$$

The empirical relation (27) suggests that the arbitrary functions g and f in (26) are the square-root function and its reciprocal, respectively, provided that the electric field intensity (V/d) be taken as a constant value and assuming the permittivity ε, the spacing d and the time T (in which the measurements are made) are relatively constant, To give the reader a glimpse of the influence of the area A of sharp edges on the corona discharge current I_{cd}, we present a graphic relation based on Eq. (27) as shown in Figure (1). It can be seen that I_{cd} tend to slowly increase with the increase of the area of the sharp edge A.

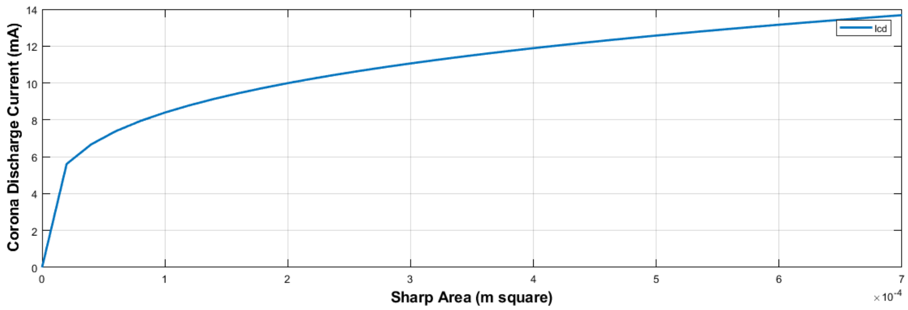

Fig. (1). Result of 6-kV/cm electric field and corona discharge current

Another simulation has been carried out to visualize the impact of particular parameters on detrimental ozone generation. Bo et al. [24] that the general arbitrary relation relating the dimensionless products in Eq. 16 can be formulated as a double-monomial relation of the form:

$$\left(\frac{r\mu}{\varepsilon(V-V_i)}\right) = k_1\left(\frac{a}{d}\right)^{k_2}\left(\frac{V}{V-V_i}\right)^{k_3} \quad (28)$$

The validity of (28) through its successful fitting to experimental results. Employing a linear regression method, Bo et al. [24] found the values of k_1, k_2 and k_3 in (28) to be $3x10^6, 2, -1$ respectively, and hence Eq. 28 can be re-expressed as follows

$$r = 3x10^6\left(\frac{a}{d}\right)^2\left(\frac{V}{V-V_i}\right)^{-1} \quad (29)$$

Using formulation of V_i in [24] and then substituting into Eq. 29, we can get the formula of ozone generation rate per unit length of wire r as follows

$$r = \frac{0.133a^2\left[V-3x10^3 a\left(1+0.3\sqrt{\frac{10}{a}}\ln\left(\frac{4d}{\pi a}\right)\right)\right]^2}{Vd^2} \quad (30)$$

To visualize the impact of parameters a, d and V for the ozone generation rate per unit length of wire r, we present a graphic representation of Eq. 30 as shown in Figure (2). It can be seen that the ozone generation rate per unit length of wire r increases with the rising of the Applied Voltage V.

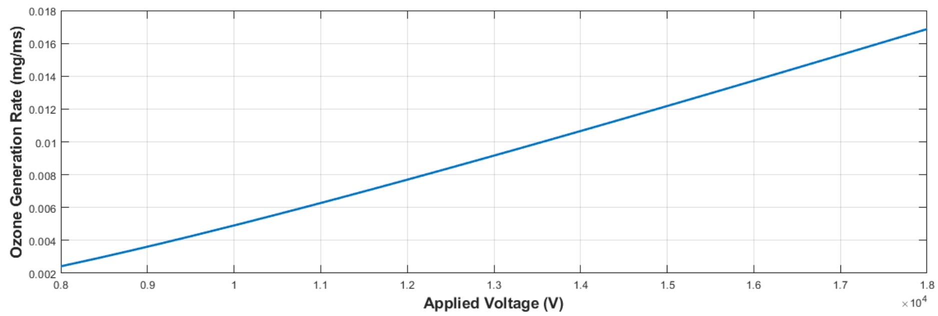

Fig. (2). Result of ozone generation rate per unit length of wire based on predicted values

5. Conclusions

Using dimensional analysis, we can determine the correlation and the mutual effects among several physical quantities. In this paper, we determined the dimensionless products pertinent for several corona-related phenomena including: (a) corona discharge current, (b) detrimental ozone generation by negative wire-to-plate corona discharge in both dry and humid air, and (c) the small current regime for rod-plane geometry in air. We used three computational methods: namely Gauss-Jordan Elimination, explicit matrix inversion and the method of fundamental solutions. Moreover, we contributed a novel explanation (reinforced with ample diverse examples) for the issues of bases, regimes and independent sets of dimensionless products. Finally, we visualized the results of our study of corona discharge to demonstrate the influence of several parameters for particular regime variables.

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الانتفاع بتحليل الأبعاد في دراسة التفريغات الهالية للشحنات الكهربية

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ملخص البحث. تقدم ورقة البحث هذه محاولات عدة للانتفاع بتحليل الأبعاد في دراسة التفريغات الهالية للشحنات الكهربية. نهدف إلى تحديد المضروبات عديمة الأبعاد المتعلقة بدراسة بعض الظواهر المتعلقة بالتفريغات الهالية مثل (أ) تيار التفريغ الهالي، (ب) التوليد الضار لغاز الأوزون بواسطة التفريغ الهالي من السلك السالب إلى اللوح في كلا من الهواء الجاف والهواء الرطب، (ج) نظام التبار الصغير لتشكيلة القضيب والمستوى في الهواء. نوضح ونقارن عدة طرائق حسابية لمعالجة منظومة الأبعاد بهدف توليد المضروبات عديمة الأبعاد المذكورة. كذلك نسهم توضيحا مبتكرا (مدعوما بأمثلة متنوعة كافية) لقضايا القواعد الإسنادية والنظم والمجموعات المستقلة من المضروبات عديمة الأبعاد.