Journal of Engineering and Computer Sciences
Qassim University, **Vol. 14, No. 1, pp. 37-63 (January 2021 /Rabi II 1441H)**

Reliability Evaluation of an Electric Power System Utilizing Fault-Tree Analysis

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(Received 28/12/2020, accepted for publication 10/6/2021)

Abstract. As a large and complex system, an electric power system may be subject to a number of disturbances which can cause full or partial failure in its operation. This premise warrants the inclusion of reliability evaluation in the planning process of a power system. Among the several available reliability evaluation techniques, fault-tree analysis is definitely considered a highly suitable one, particularly for dealing with the inherent complexity of power systems. This paper applies of fault-tree analysis for a partial reliability evaluation of a specific power system known as the Roy Billinton Test System (RBTS). The proposed study is to calculate the reliability of power delivery to the largest load point of the RBTS. The steps carried out in the analysis include fault-tree construction, qualitative analysis by establishing the minimal cutsets, and quantitative analysis by calculating reliability utilizing the data of failure probability of each component of the system. The results presented in this study include failure probabilities for the minimal cutsets, failure probability of power delivery to the system's largest load and the corresponding reliability, as well as the importance measures of various components. The analysis produces a set of twenty minimal cutsets for the power delivery to the largest load point in the RBTS, with a corresponding numerical value of 0.999597 of system partial reliability for the assumed component reliabilities. The importance analyses utilizing Fussell-Vesely importance, risk achievement worth, and risk reduction worth finds that BUS3 is the most important system component for the power delivery to the largest load, while transmission line L5 is the least important system component.

***Keywords*:** Reliability evaluation, Fault-tree analysis, Electric power system, Minimal cutsets, Load point, Importance measure.

1. Introduction

Electric power systems are arguably among the largest and the most complex systems ever built by humans. Their architectures and types greatly vary in different places, but their ultimate goal is the same, that is to deliver power to the consumers in a reliable and economical manner. Economy and reliability considerations are indeed competitive, but can be balanced in many circumstances. A mandatory task is to incorporate the reliability constraint in the planning process in order to achieve successful operation. A necessary step to achieve this task is to perform extensive reliability evaluation of any proposed power system.

Many significant reliability evaluation techniques exist in practice. They can be categorized into two groups, namely the group of simulation techniques and that of analytical techniques. While each of these technique groups possesses certain advantages and disadvantages, the latter group of techniques is relatively simpler, especially if the system being evaluated is not very complex [1]. Among the existing reliability evaluation techniques, fault-tree analysis (FTA) is often mentioned as the one of the most useful tools for the reliability evaluation of complex systems. Being first developed in 1961 for purposes related to missile launch systems and nuclear reactor safety, FTAs have been enjoying a dramatic increase in their applications since then [2].

Examples of FTA applications can be found in computer security modeling [3], safety assessment of nuclear power plants [4], and reliability assessment of electric railway substations [5]. Applications in power engineering include reliability assessment of the power distribution grid [6], uninterruptable power supply (UPS) systems [7], power system protection [8], thermal power plants [9], wind energy systems [10], smart grids [11], hybrid renewable energy systems [12], and bulk power systems in general [13].
The contribution of this paper is the application of FTA for reliability evaluation of the Roy Billinton Test System, which -at the authors' best knowledge- has not been discussed in any previous studies yet. In this paper, FTA is applied to compute a measure of partial reliability of a power system by considering the failure of power delivery to the largest load point as the top event. This means that the system reliability is seen solely from the viewpoint of this largest load point. However, FTA can conveniently be applied to the failure of power delivery to all other load points in order to calculate a measure of the total reliability of the system.

2. Method and Material

This section starts with a brief discussion on the basics of power system reliability. It is then followed by a short review on FTA and how it is applied to power systems. The description of the test system used in this study and the associated fault-tree construction then concludes this section.

2.1 Definition of Power System Reliability

A generally agreed-upon concept in reliability circles is that failure of a (rather good) system is easier to define than non-failure. Departing from this concept, we find it more convenient to define power system reliability R_{PS} in terms of its unreliability U_{PS}. That is, we express the reliability as

$$R_{PS} = 1 - U_{PS} \tag{1}$$

The system unreliability U_{PS} of a power system of different load points is defined by the following equation [13],

$$U_{PS} = \sum_{i=1}^{N_L} U_i \frac{K_i}{K} \tag{2}$$

where U_i is the unreliability (also known as the failure probability) of the power delivery to load point i, N_L is the number of load points, K_i is the size of load point i (in megawatts (MW)), and K is the total load of the power system (also in MW). Thus, the dimensionless ratio K_i/K can be considered as a weighting factor for the unreliability of load i. Equation (2) clearly shows that the calculation of the overall system unreliability is carried out through the calculation of the unreliability of power delivery to each load point. The largest load points in the system contribute most to the overall system unreliability, while the contributions from the smallest load points are the least. The role of FTA, as will be described in the subsequent sub-sections, is to investigate these "individual or partial unreliabilities", each of which represents the system unreliability from the viewpoint of a specific load point.

2.2 A Brief Review on FTA and Its Applications

The essential core of FTA is the transformation of a certain physical system into a functional logic diagram called "a fault tree". The diagram is arranged in such a way as to represent a flow of logic from the bottom to the top of the tree. The tree top is the "top event" of interest and the remaining parts of the tree are the specific events that lead to this top event. As the name implies, the top event represents a certain failure (or in a more general sense, an undesired event) occurrence. Fault trees are constructed by using a set of event symbols and a set of logic symbols. The event symbols consist of rectangle, circle, and triangle, while the logic symbols can be AND gate and OR gate. The logic symbols or gates systematically connect two or more events, with the exception of the NOT gate, which inverts a certain single event. [14]–[18]. An immediate extension of the concept of a fault tree involves multiple top events in a complex logic diagram called a multiple-output fault tree or a fault forest [19]–[21].

Fault tree analysis involves several steps, which are arranged in the following sequence: (i) system definition, (ii) fault tree construction, and (iii) analysis. For the power system that we observe, each of these steps are described as follows.

(i) System Definition

This step basically involves defining the top event and the remaining events, defining how these events are interconnected with each other, and defining the failure state of each component. These will shape the fault tree that will be constructed. In our study, we define the failure of the power supply to a specific load point as a top event. It will be easily seen that the FTA for a specific load point can be conveniently constructed by way of analogy to other load points. The failure of supply to a load point can be caused by one or several events, such as the failure of a bus, the failure of a generating unit, and/or the failure of a transmission line. For all these components, we use the simple two-state (up/down) model. As an example, a generator is considered up if it can supply a specific designated power level. Otherwise, it is deemed down. No other conditions are considered here. All these events will interact with each other through OR and AND logic gates. No NOT gate is needed in the present analysis.

(ii) Fault Tree Construction

It is noted again here that a fault tree depicts the structure of contributing events, rather than components. Fault tree construction is the most complicated and the most time-consuming and error-prone step in FTA. In examining an event, one should identify all possible events that cause it. In this study, since we evaluate only one top event, namely, the failure of power delivery to the system’s largest load point, there will be one (single-output) fault tree only.

(iii) Analysis

The analysis is carried out based on the constructed fault tree. There are basically two types of analysis, namely qualitative and quantitative analysis. One may opt to choose either of these types, or to use both types, depending on the structure of the fault tree evaluation. In this study, both types of analysis will be considered and carried out. In qualitative analysis, we identify a minimal cutset representing a combination of component failures which results in system failure, i.e., a condition in which the load point is not adequately supplied. In quantitative analysis, we aim to calculate the probability of the top event, i.e., to calculate the system unreliability under the specified conditions. For this purpose, the top event structural representation in terms of basic events is required. Qualitative analysis through minimal cutset identification is one way to accomplish it. For ease of doing analysis, each minimal cutset is assigned a unique number i, $i = 1,2, \dots, m$, where m is the number of the minimal cutsets of the system.

In quantitative analysis, the failure probability of minimal cutset i is calculated as follows [13]

$$U_{mcs,i} = \prod_{k=1}^{n_i} U_{k,i} \tag{3}$$

where $U_{mcs,i}$ is the failure probability of minimal cutset i, n_i is the number of events constituting minimal cutset i, and $U_{k,i}$ is the failure probability of event k in minimal cutset i. Note that, the term "event" in the minimal cutset refers to the failure of a single component in the system. An upper bound on the unreliability can then be computed by summing the failure probabilities of all minimal cutsets. This bound is very tight when these failure probabilities are very small, and is usually referred to as the Boole-Bonferroni first bound or inequality [22]–[25]. An exact value of the unreliability might be obtained via the disjointing procedure outlined in [3], [8]. Once the unreliability is obtained, the reliability can easily be calculated using (1).

Using the data of the minimal cutsets, we also carry out analysis on determining the importance of the system's components forming the basic events. To do so, three importance measures are calculated, namely Fussell-Vesely importance, risk achievement worth (RAW), and risk reduction worth (RRW) [13], [26]–[30]. The Fussell-Vesely importance measure I^{FV} is the probability that at least one minimal cutset containing component k fails, given that the system is failed. Thus, the Fussell-Vesely importance for component k is approximated by

$$I^{FV}(k) \approx \frac{\sum_{m=1}^{n_k} U_m^k}{U_{LOAD1}} \tag{4}$$

where U_{LOAD1} is the unreliability of the power delivery to the pertinent load point, designated here as LOAD1 (computed as the sum of failure probabilities of all minimal cutsets for power delivery to the pertinent load point) and U_m^k denotes the failure probability of minimal cutset m among those containing the component k. The RAW measure of the component k is the measure of relative increase in the system unreliability when this component fails, i.e., the unreliability of the component k is set to 1. It is given by

$$RAW_k = \frac{U_{LOAD1}(U_k = 1)}{U_{LOAD1}} \tag{5}$$

where $U_{LOAD1}(U_k = 1)$ the unreliability of the power delivery to the pertinent load point when the component k fails, and U_k is the unreliability of the component k. The third measure to be considered herein, i.e. RRW, is the relative reduction of the system reliability when the component k functions, that is when its unreliability is set to 0. This is expressed as

$$RRW_k = \frac{U_{LOAD1}}{U_{LOAD1}(U_k = 0)} \tag{6}$$

The method that has just been described is applied to an educational test system designated as the Roy Billinton Test System (RBTS) [31]–[33]. The system is sufficiently small to conduct a large number of reliability studies with reasonable solution times. The single line diagram and data of the system can be seen in Appendix.

2.3 The Calculation of the Unreliability of Power Plants

The RBTS consists of two independent power plants, each of which comprises a specific number of generating units. As shown in Figure A1, the power plants G1 and

G2 consist of four and seven units, respectively. Since each of these power plants is a threshold system [34], [35], their unreliability calculation cannot be carried out in a straightaway fashion, such as the one used in handling logically series or parallel systems. Expressing each of the successes of G1 and G2 as a switching function of component successes $S(\mathbf{X}) = S(X_1, X_2, \dots, X_n)$, we find that both G1 and G2 thus satisfy the following threshold system criterion [34], [35],

$$S(\mathbf{X}) = 1 \text{ iff } F(\mathbf{X}) = \sum_{i=1}^{n} W_i X_i \geq T \tag{7}$$

The number of components n represents the number of constituting generating units, while the weight W_i denotes the capacity size of unit i. The threshold T is the load demand which should be satisfied. Because our concern in this study is the power delivery to the RBTS's largest load point, i.e., the load point of 85 MW, thus both G1 and G2 are assumed to be threshold systems each with each a threshold value of 85. We base our calculations on an implicit assumption that other load points do not compete for a share of the power generated.

To calculate the unreliability of G1, we utilize the aid of the Karnaugh map [34], [35]. We first express the power generation of plant G1 as the following pseudo-Boolean function (in which the $'+'$ operator retains its standard meaning of arithmetic addition),

$$F_1(\mathbf{X}) = 40X_1 + 40X_2 + 20X_3 + 10X_4 \tag{8}$$

Figure 1 shows two Karnaugh map representations, one for $F_1(\mathbf{X})$ in (8) and another for $E\{S(\mathbf{X})\}$ obtained via (7) in conjunction with (8). The real "generation" entries whose values are greater than 85 in the pseudo-Boolean map in Figure 1(a) are transformed into binary values of 1 in the probability map in Figure 1(b). Note that the notations "R" and "U" are used in Figure 1(b) to denote the reliability and the unreliability of generating units in G1.

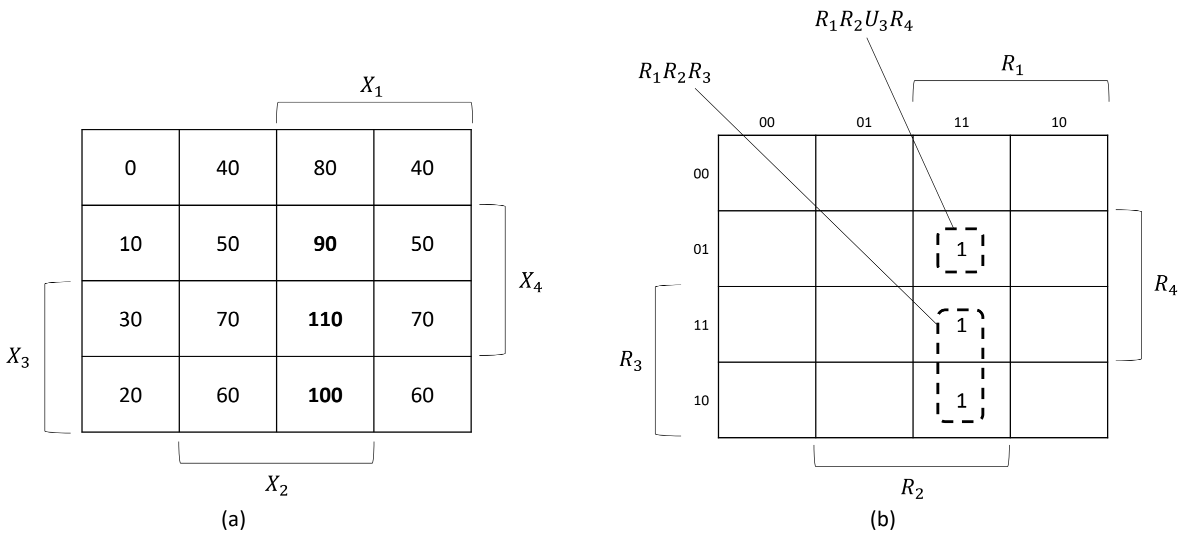


Fig 1. Karnaugh map for the calculation of the unreliability of G1: (a) the pseudo-Boolean function $F_1(\mathbf{X})$; (b) the probability map for $E\{S(\mathbf{X})\}$ (Karnaugh map with disjoint loops)

From the map in Figure 1(b), the reliability of G1 can be expressed as follows,

$$R_{G1} = R_1R_2R_3 + R_1R_2U_3R_4 \tag{9}$$

Substituting the units' unreliability data in Table A1 to (9) results in a value of 0.9404, which corresponds to the unreliability value of 0.0596.

For G2, we employ the notation **Y** to differentiate its component successes from those of G1. Hence, the power generation of plant G2 is expressed as

$$F_2(\mathbf{Y}) = 40Y_1 + 20Y_2 + 20Y_3 + 20Y_4 + 20Y_5 + 5Y_6 + 5Y_7 \tag{10}$$

The calculation of the unreliability of G2 is clearly more complicated since it involves seven variables representing seven generating units. To cope with this complexity, we employ the Signal Flow Graph based on the recursive algorithm described in [34]–[36]. Based on threshold system terminology, the system expressed by (10) can be re-written as $H(7; p; 40, 20, 20, 20, 20, 5, 5; 85)$ [35]. Its reliability can then be obtained the following recursive relations

$$R(n;\mathbf{p};\mathbf{W};T) = q_i R(n-1;\ \mathbf{p}/p_i;\ \boldsymbol{W}/W_i;\ T) \\ + p_i R(n-1;\ \mathbf{p}/p_i;\ \boldsymbol{W}/W_i;\ T - W_i) \tag{11}$$

$$R(n;\mathbf{p};\mathbf{W};T) = \begin{cases} 1 & \text{if } 0 \geq T & (12a) \\ 0 & \text{if } \sum_{i=1}^{n} W_i < T & (12b) \end{cases}$$

The best policy to decompose the system is by arranging the weights in a descending order starting from the largest weight, whose sequence is shown in Figure 2 to Figure 4. The Signal Flow Graph that implements the best policy for the pertinent problem is shown in Figure 4, in which the black nodes are source nodes of value 1 and the white ones are source nodes of value 0. Note that in the best policy, we decompose the system success with respect to the component success of the largest weight first.

	∅	[5]	[5, 5]	[20, 5, 5]	[20, 20, 5, 5]	[20, 20, 20, 5, 5]	[20, 20, 20, 20, 5, 5]	[40, 20, 20, 20, 20, 5, 5]
T = -15								
T = 0								
T = 5								
T = 15								
T = 25								
T = 35								
T = 45								
T = 55								
T = 65								
T = 75								
T = 85								

Fig. 2. Distribution of nodes in the two-dimensional plane of threshold versus weights for plant G2. A blue cell should contain a non-source node expressed recursively via (11), and hence should have two arrows incident on it that emanate from nodes in the column to its left. Other cells represent source nodes: green cells containing nodes of unity values according to the condition in (12a), and yellow cells containing nodes of zero values according to the condition in (12b)

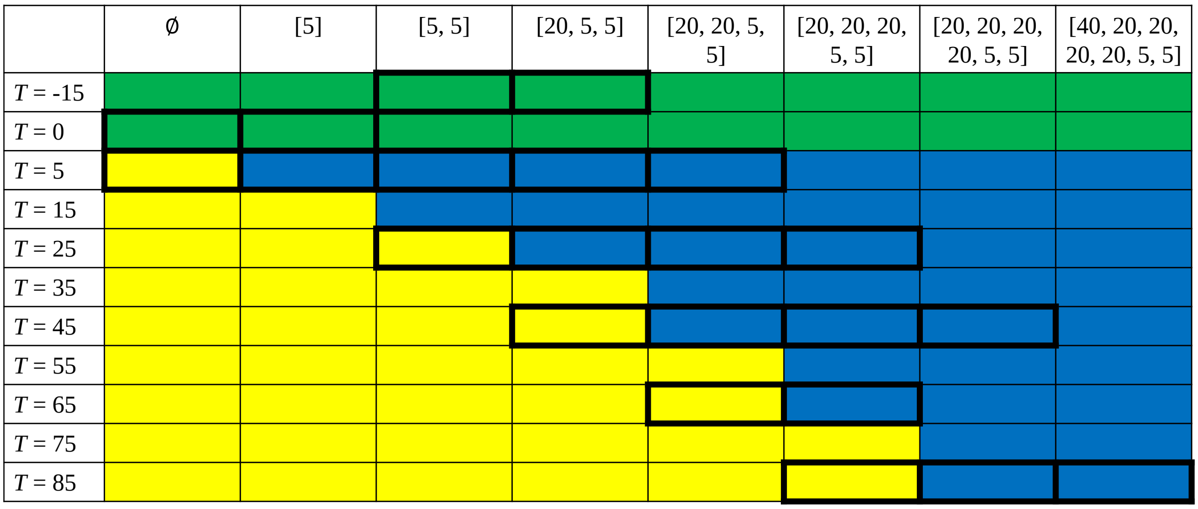


Fig. 3. Locations of all nodes which are involved in the construction of Signal Flow

Graph for plant G2. A blue cell in a certain column is replaced by exactly two nodes in the column to its left, one at the same horizontal level, and another at a level higher by an amount equal component w.r.t. which expansion is performed. Processing terminates at a green cell of unity value or a yellow cell of zero value.

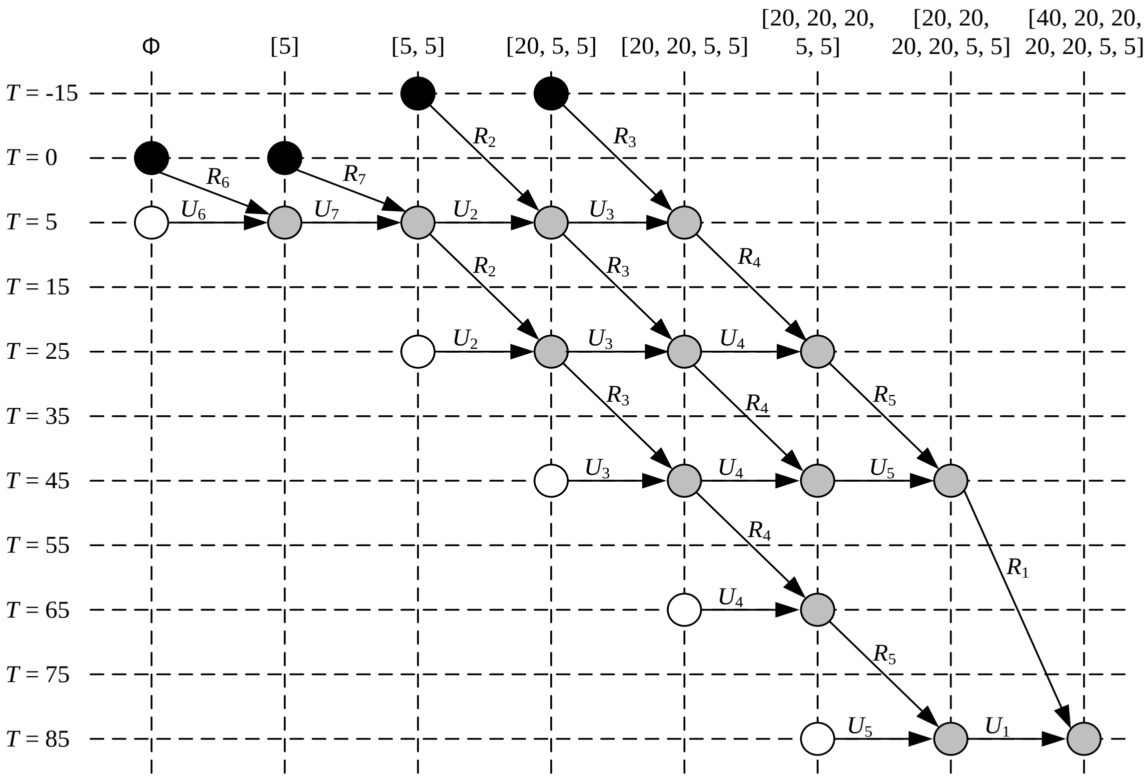


Fig. 4. Best policy for the Signal Flow Graph to represent the reliability of G2 when decomposition is with respect the largest weights first. This graph is constructed based on the node locations shown in Figure 3.

Once the Signal Flow Graph is established, the Karnaugh map for G2 is arranged using a procedure as that for G1. The role of the Signal Flow Graph is to help identify the loops in the probability map. The entire maps for G2 are shown in Figure 5. Note that the notations R and U in Figure 5(b) are different from those in Figure 1(b) even for those with the same subscripted numbers. This is because the notations in the former indicate the hydro units in G2, while those in the latter denote the thermal units in G1. The reliability of G2 is the sum of all 18 terms in the probability map in Figure 5(b). Using the data in Table A1, we achieved the reliability value of 0.9988 for G2, which corresponds to the unreliability value of 0.0012.

3. Results and Discussion

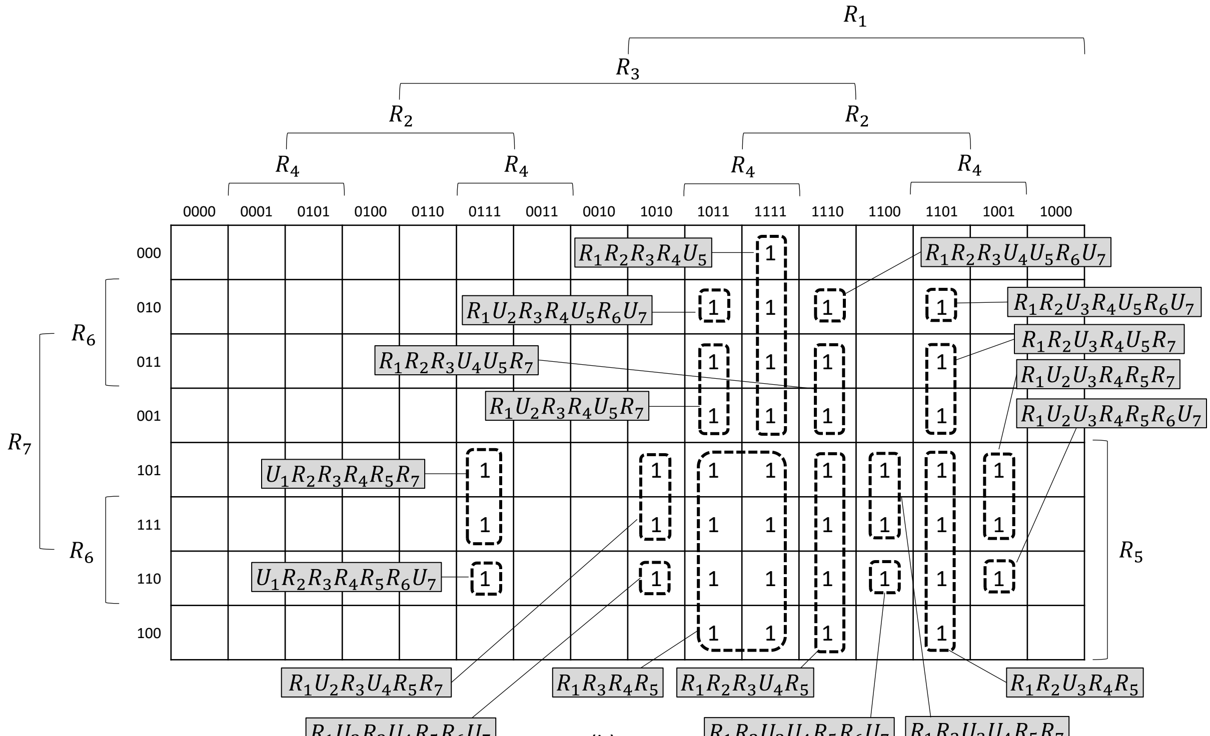
In this section, we present the results of FTA application along with a discussion. The results are presented based on the steps described in Sub-Section 2.2.

3.1 Fault Tree Construction

As previously described, the top events for the FTA in this study are the failure of power supply to load points in the system. Since there are five load points in the RBTS, as shown in Figure A1, there are five possible fault trees, which are the fault trees associated with the failure of power delivery to LOAD1, LOAD2, LOAD3, LOAD4, and LOAD5, respectively. In this paper, however, we only consider and construct the first one, since LOAD1 is the most important and the largest load point representing 45.95% of the total load. Nevertheless, the method for constructing this first fault tree can easily be applied to develop the other fault trees. Therefore, the resulting total probability of the studied system is restricted to be that from the viewpoint of LOAD1.

0	20	40	20	40	60	40	20	60	80	**100**	80	60	80	60	40
5	25	45	25	45	65	45	25	65	**85**	**105**	**85**	65	**85**	65	45
10	30	50	30	50	70	50	30	70	**90**	**110**	**90**	70	**90**	70	50
5	25	45	25	45	65	45	25	65	**85**	**105**	**85**	65	**85**	65	45
25	45	65	45	65	**85**	65	45	**85**	**105**	**125**	**105**	**85**	**105**	**85**	50
30	50	70	50	70	**90**	70	50	**90**	**110**	**130**	**110**	**90**	**110**	**90**	70
25	45	65	45	65	**85**	65	45	**85**	**105**	**125**	**105**	**85**	**105**	**85**	65
20	40	60	40	60	80	60	40	80	**100**	**120**	**100**	80	**100**	80	60

(a)



(b)

Fig. 5. Karnaugh map for the calculation of the unreliability of G2: (a) the pseudo-Boolean function, $F_2(\mathbf{Y})$; (b) the probability map (Karnaugh map with disjoint loops)

Figure 6 shows the starting part of the fault tree, in which the top event is the failure of power delivery to LOAD1. Its construction goes in line with the development of the functional tree of power flow paths. It starts from BUS3, so a failure of BUS3 is deemed as an immediate basic event. This means that if BUS3 fails, there is no power delivery to LOAD1. There is another event which can fail the power delivery to LOAD1, which is the failure of power delivery to BUS3. From the single line diagram in Figure 1, three sources of power to BUS3 can be identified: the connection between BUS3 and BUS1, the connection between BUS3 and BUS4, and the connection between BUS3 and BUS5. All these three sources have to fail simultaneously to make power delivery to BUS3 fails, which means that an AND gate should be used to connect them.

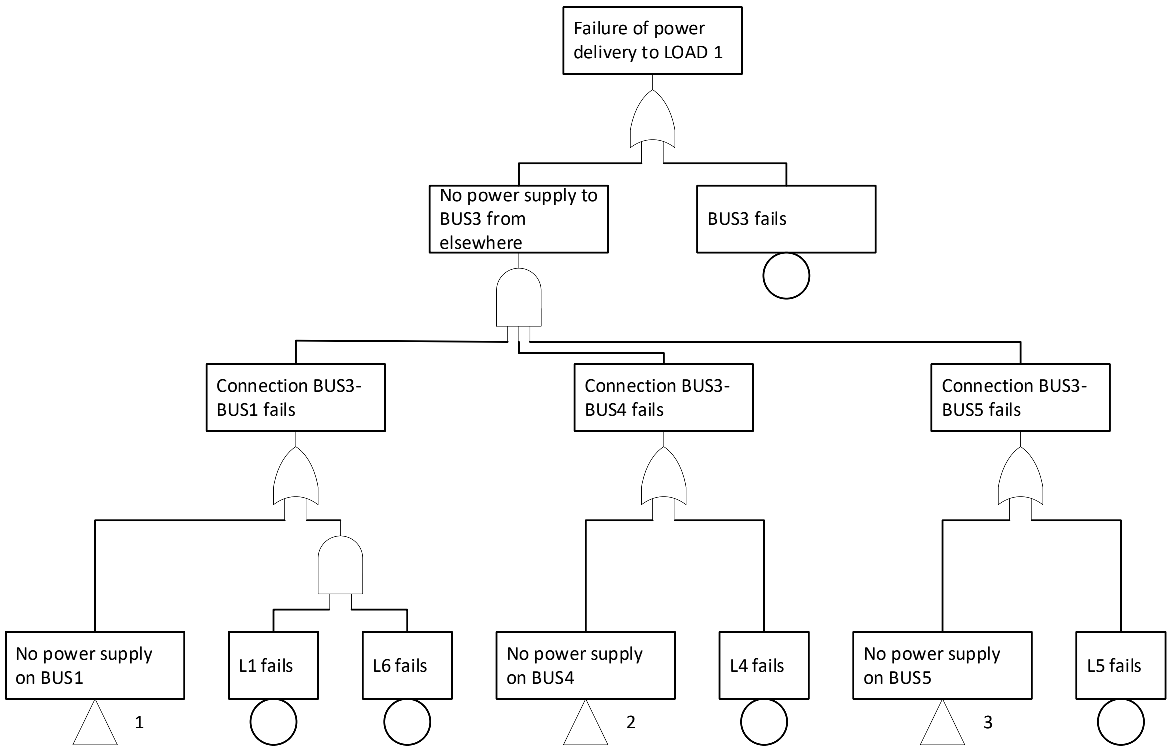


Fig. 6. Fault tree for the failure of power delivery to LOAD1 as a top event.

There are two sources that establish the successful operation of each of these three connections. Connection between BUS3 and BUS1 is successful if there is power supply on BUS1 and either L1 or L6 is successful. It means that this connection fails if either of these two success requirements is not fulfilled, which indicates that an OR gate should be used to connect the corresponding failures. According to De' Morgan rule, we denote the simultaneous failure of L1 and L6 as a basic event, which complements the event of success of either of them. Similarly, the connection between BUS3 and BUS4 fails if either there is no power supply on BUS4 or L4 fails. Also, the connection between BUS3 and BUS5 fails if either there is no power supply on BUS5 or L5 fails. The events of "L4 fails" and "L5 fails" are considered as basic events. The events other than the basic events at this stage require further development and are marked with triangles and specific annotations to indicate that they are continued in the next figures.

Figure 7 shows the continuation of the fault tree at the event “no power supply on BUS1”. This failure is basically an OR event, since it occurs when either BUS1 itself fails or there is no power supplied by neighboring sources. The failure of BUS1 is a basic event here. On another side, the absence of power supply to BUS1 can also take place when both G1 and the connection between BUS1 and BUS2 fails. Thus, this is an AND event, where the former is a basic event and the latter is an OR event. Moving to the connection between BUS1 and BUS2, we can see that the failure of this connection is caused by the failure of BUS1 or the failure of generator G2. The current branch of the fault tree in Figure 7 ends here because the generator G2 is the last source of power for this function tree of power flow.

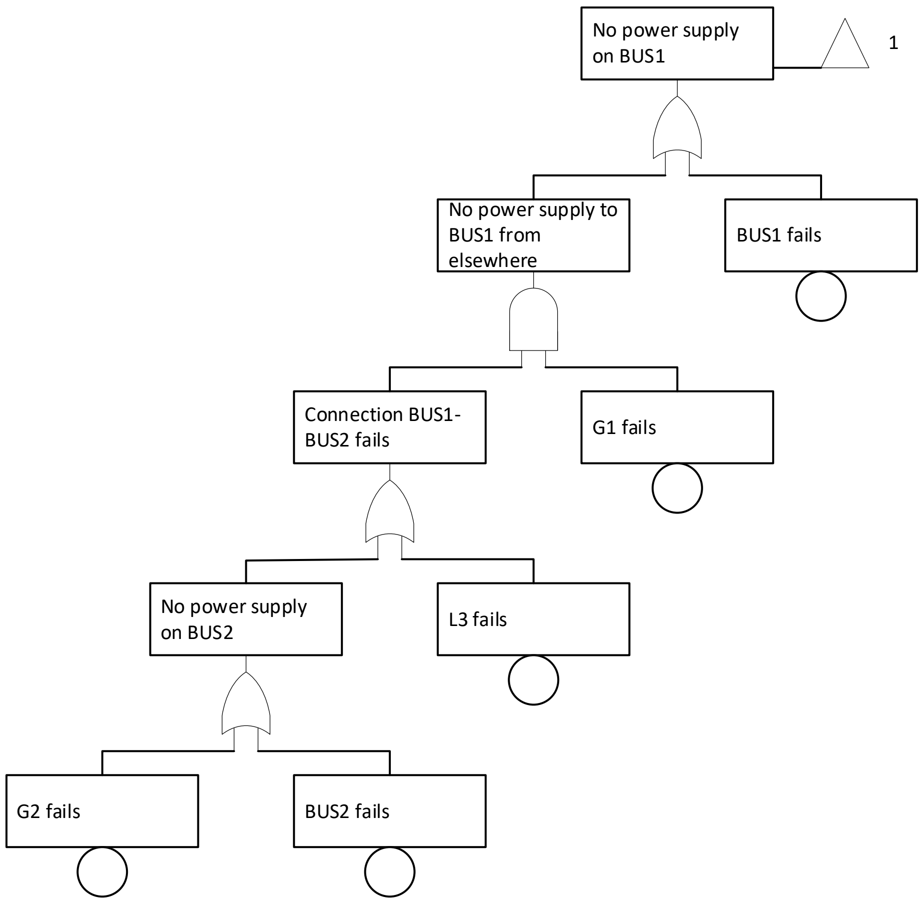


Fig. 7. Fault tree for the failure of power delivery to LOAD1 (continuation 1)

The other branches of the fault tree associated with the failure of power delivery to LOAD1 are developed in a similar way as we discuss now. These other branches or continuations of the main fault tree in Figure 7, which are the continuations at the points 2 and 3, are shown in Figure 8 and Figure 9, respectively.

For continuation 2, the power is failed to be supplied to BUS4 if either BUS4 itself fails or the connection between BUS4 and BUS2 fails, thus this is an OR event. The path towards BUS5, and consequently to BUS6, is neglected since there is no generator beyond these two buses. The connection between BUS4 and BUS2 fails if either both the transmission lines L2 and L7 fail or there is no power supply on BUS2. Consequently, this is an OR event containing an AND sub-event of the transmission lines L2 and L7 failures. The failures of such transmission lines are thus considered as basic events at this stage. The tree continues until reaching BUS1 as the bus to which the last generator is connected. The failure of the power supply to BUS1 is an OR event, which can occur if either BUS1 itself fails or generator G1 fails. This branch of the fault tree ends here, with the last-mentioned two events deemed basic events. For continuation 3 (Figure 9), the starting event is the failure of power supply to BUS5, which can occur if either BUS5 itself fails or the connection between BUS5 and BUS4 fails. The failure of BUS5 is a basic event at this stage, while the remaining part of this continuation is exactly the same as that of continuation 2 since the power flows through the same path.

3.2 Qualitative Analysis

Qualitative analysis is made by establishing the system minimal cutsets representing the combinations of component failures which can fail the power delivery to a certain load, which is LOAD1 in the present case. Table 4 presents the minimal cutsets for the fault tree associated with the failure of power delivery to LOAD1, in which there are twenty minimal cutsets in total. The single minimal cutset that comprises one basic event only is the failure of BUS3. This means that the failure of BUS3 alone is sufficient

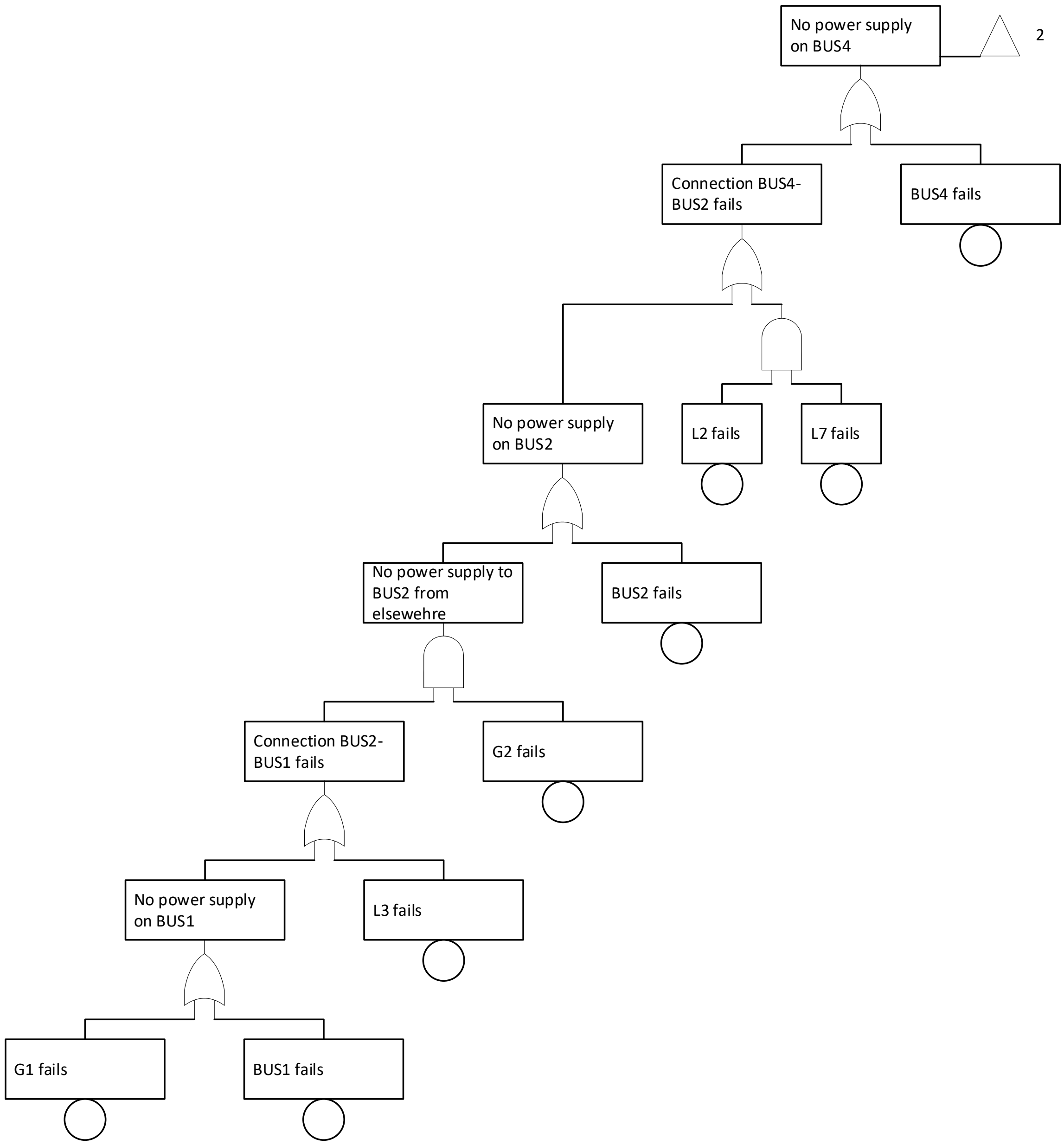


Fig. 8. Fault tree for the failure of power delivery to LOAD1 (continuation 2)

to make the power delivery to LOAD1 fails. The remaining minimal cutsets might be categorized as six double-event minimal cutsets, six triple-event minimal cutsets, and seven quadruple-event minimal cutsets. For these three categories, two, three, or four simultaneous failure events are needed to fail the power delivery to LOAD1, respectively. Using the same logic, one can easily list the minimal cutsets for the remaining four load points in the RBTS.

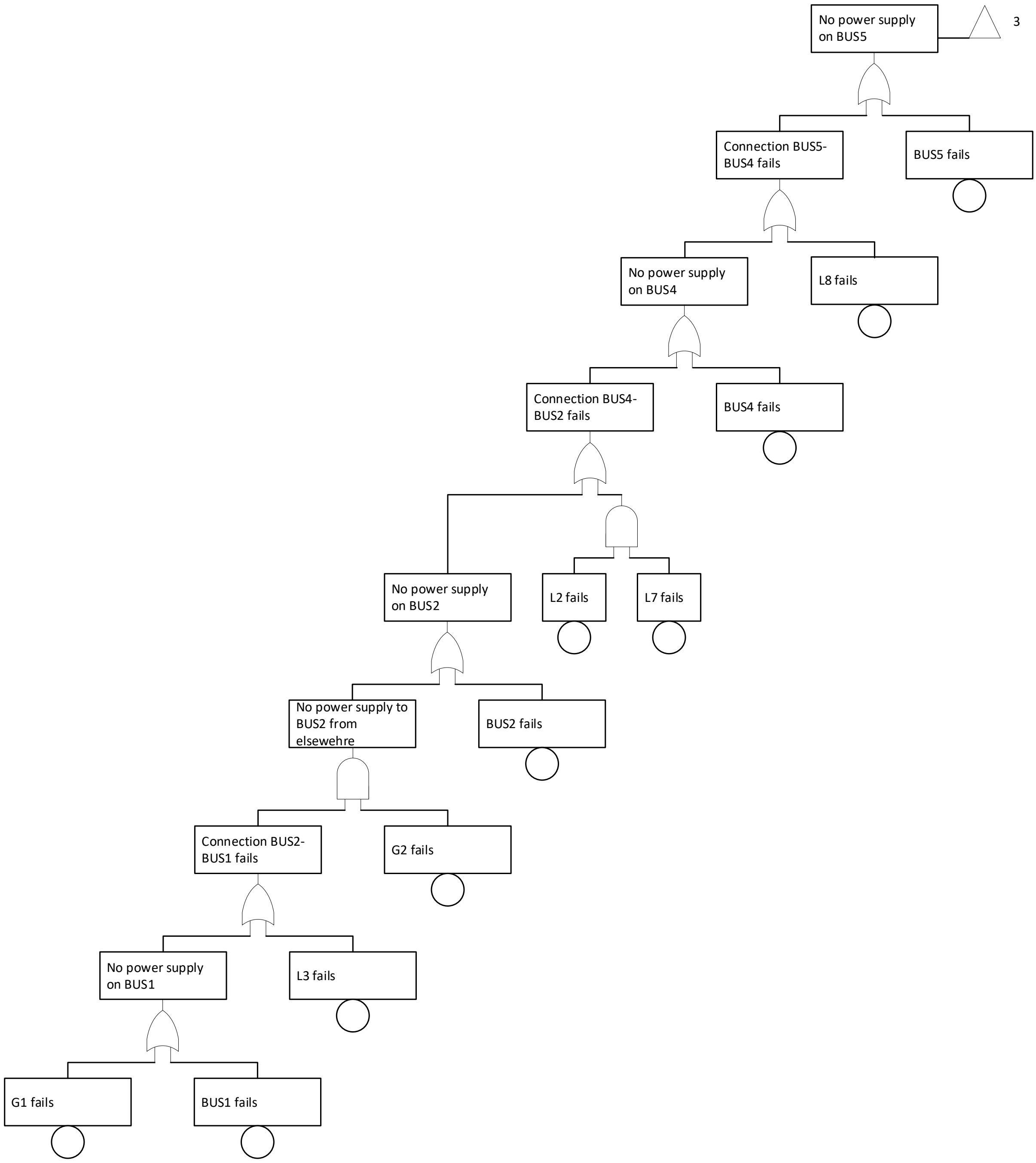


Fig. 9. Fault tree for the failure of power delivery to LOAD1 (continuation 3)

Table 1. Minimal cutsets for the failure of power delivery to LOAD1

No.	Event 1	Event 2	Event 3	Event 4
1	BUS3			
2	G1	BUS1		
3	G1	G2		
4	G1	BUS2		
5	BUS1	G2		
6	BUS1	BUS2		
7	BUS1	BUS4		
8	G1	L3	BUS4	
9	BUS1	L2	L7	
10	BUS1	L4	L8	
11	BUS1	L4	L5	
12	BUS2	L1	L6	
13	L1	L6	BUS4	
14	G1	L3	L2	L7
15	G1	L3	L4	L8
16	G1	L3	L4	L5
17	G2	L1	L6	L3
18	L1	L6	L4	L5
19	L1	L6	L4	L8
20	L1	L6	L2	L7

3.3 Quantitative Analysis

The purpose of fault tree quantitative analysis is to compute the probability of the system's minimal cutsets (which gives the system unreliability from the viewpoint of power delivery to LOAD1) and other system's important measures. The probability of the minimal cutsets for the power delivery to LOAD1 is calculated using (3) and is presented in Table 5. As can be seen in Table 5, minimal cutset 1, i.e. the failure of BUS3, is the most significant one with the highest failure probability. This is because this minimal cutset comprises only one component, thus making it the most vulnerable cutset for the power delivery to LOAD1. On the other hand, minimal cutsets which involve the largest number of components are those with the lowest failure probabilities. Minimal cutsets 15 to 20 fall into this category, with the cutsets numbered 18 and 19being placed as the bottommost.

Table 2. The failure probabilities of the minimal cutsets for the power delivery to LOAD1

Minimal cut set no	**Failure Probability**	**%**	**Minimal cut set no**	**Failure Probability**	**%**
1	3.00×10^{-4}	73.392177	11	3.63×10^{-10}	0.000089
2	1.79×10^{-5}	4.374174	12	5.10×10^{-7}	0.124767
3	7.15×10^{-5}	17.496695	13	8.67×10^{-10}	0.000212
4	1.79×10^{-5}	4.374174	14	8.91×10^{-9}	0.002179
5	3.60×10^{-7}	0.088071	15	3.32×10^{-10}	0.000081
6	9.00×10^{-8}	0.022018	16	3.32×10^{-10}	0.000081
7	9.00×10^{-8}	0.022018	17	1.60×10^{-11}	0.000004
8	8.22×10^{-8}	0.020121	18	3.50×10^{-12}	0.000001
9	9.75×10^{-9}	0.002385	19	3.50×10^{-12}	0.000001
10	3.30×10^{-7}	0.080731	20	9.39×10^{-11}	0.000023

Once the system's minimal cutsets are established, the system unreliability from the viewpoint of the power delivery to LOAD1 can then be expressed as an OR event whose inputs are all minimal cutsets listed in Table 4. A tight upper bound on the probability of occurrence of this event is the sum of the failure probability values listed in Table 5. This bound is 4.09×10^{-4}. Therefore, the RBTS reliability from the viewpoint of LOAD1 is almost 0.999591, an excellent value. From the data of the minimal cutsets in Table 4 and Table 5, the importance of each basic event/components is also computed. As described in Sub-section 2.2, there are three importance measures to be calculated herein, namely Fussell-Vesely importance, RAW, and RRW. The results are shown in Table 6.

Table 3. Importance measures of component/basic events for the power delivery to LOAD1

No	Basic Event / Component	Minimal Cut Set No	Fussell-Vesely Importance	Risk Achievement Worth	Risk Reduction Worth
1	BUS1	2,5,6,7,9,10,11	0.0458948	153.9369	1.048103
2	BUS2	4,6,12	0.0452096	148.2284	1.047350
3	BUS3	1	0.7339218	2446.6720	3.758293
4	BUS4	7,8,13	0.0004235	2.4113	1.000424
5	G1	2,3,4,8,14,15,16	0.2626751	5.1446	1.356254
6	G2	3,5,17	0.1758477	147.3639	1.213368
7	L1	12,13,17,18,19,20	0.0012501	1.7341	1.001252
8	L2	9,14,20	0.0000459	1.0080	1.000046
9	L3	8,14,15,16,17	0.0002247	1.0486	1.000225
10	L4	10,11,15,16,18,19	0.0008098	1.7354	1.000810
11	L5	11,16,18	0.0000017	1.0016	1.000002
12	L6	12,13,17,18,19,20	0.0012501	1.0014	1.001252
13	L7	9,14,20	0.0000459	1.0080	1.000046
14	L8	10,15,19	0.0008081	1.0007	1.000809

Table 6 shows a quite distinct value for the component BUS3. When BUS3 is assumed to fail, the unreliability jumps by more than 2000 times. In contrast, when it is assumed to function, the unreliability is decreased by a factor of almost 4. These high values for the three importance measures assert the significance of BUS3 for the power delivery to LOAD1. Other important components include BUS1, BUS2, and G2, although their importance measures are by far still below that of BUS3. Meanwhile, the three importance measures also give some information on the least important component, which is transmission line L5. This is indicated by the lowest values for the three measures for this component.

4. Conclusions

In this paper, partial reliability of the RBTS is investigated for the power delivery to the largest load point of the system by using fault-tree analysis. The computations are valid under different assumptions regarding the demand met by other load points. The steps carried out include the fault tree construction, qualitative analysis, and quantitative analysis. From the qualitative analysis, a set of twenty minimal cut sets can be formed, resulting in a system unreliability of 4.09×10^{-4}. This value corresponds to a system reliability of 0.999591. The subsequent analysis includes the investigation of three importance measures for all components constituting the basic events. These are Fussell-Vesely importance, risk achievement worth, and risk reduction worth. According to these measures, it is concluded that BUS3 is the most important component, while transmission line L5 is the least important one.

It is worth noting that the underlying assumption of using the first Boole-Bonferroni upper bound is justified, since all individual cutset probabilities are so small that the probability of the conjunction of two or more cutsets is truly negligible. The aforementioned assumption is so entrenched in the study of power-system reliability that many scholars tend (safely) to ignore or even forget it totally, and hence treat the upper bound and exact values of system unreliability as if they were equal or even synonymous.

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Appendix: The Roy Billinton Test System

The single line diagram of this system is shown in Figure A1, while the generating unit and reliability data are presented in Table A1. The system has two power plants G1 and G2, which are connected to BUS1 and BUS2, respectively. The former plant consists of three types of thermal units (totaling four units) and the latter consists of three types of hydro units (totaling seven units). Together, the two plants supply five load points distributed in five different buses, marked with LOAD1 to LOAD5 in Figure A1.

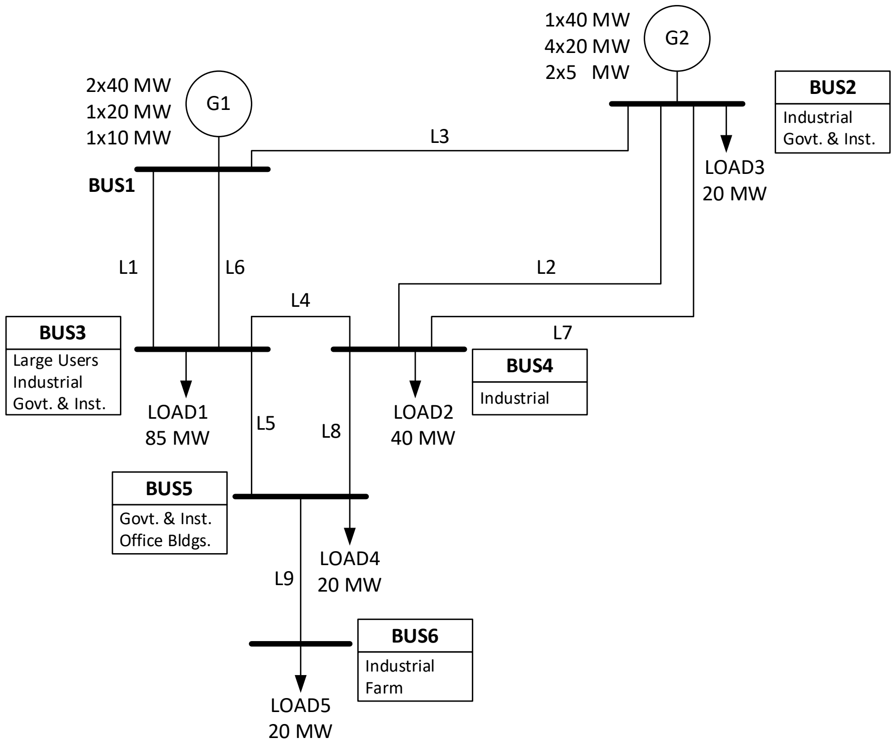

Fig. A1. Single line diagram of the RBTS

Table A1. Generating unit rating and reliability data

Unit Size (MW)	**Type**	**No. of Units**	**Bus**	**Failure Rate λ (1/yr)**	**Repair Rate μ (1/yr)**	**Failure Probability $U = \frac{\lambda}{\lambda+\mu}$**
5	Hydro	2	2	2.0	198	0.010
10	Thermal	1	1	4.0	196	0.020
20	Hydro	4	2	2.4	157	0.015
20	Thermal	1	1	5.0	195	0.025
40	Hydro	1	2	3.0	147	0.020
40	Thermal	2	1	6.0	194	0.030

Table A2 gives the transmission line lengths and outage data. The permanent outage rate of a given line is obtained using a value of 0.02 outages/year/km. Line transient outage rates are calculated using a value of 0.05 outages/year/km. The failure of a bus in this study is assumed to incorporate the bus section only, and the failure data of other station equipment, such as transformers and circuit breakers, are neglected.

Table A2. Transmission line lengths and outage data for components of the RBTS

Line	**Buses**		**Length (km)**	**Permanent Outage Rate γ (occ/year)**	**Outage Duration t (hr)**	**Failure Probability $Q = \gamma t/8760$**
	From	**To**				
1	1	3	75	1.5	10	0.0017
2	2	4	250	5.0	10	0.0057
3	1	2	200	4.0	10	0.0046
4	3	4	50	1.0	10	0.0011
5	3	5	50	1.0	10	0.0011
6	1	3	75	1.5	10	0.0017
7	2	4	250	5.0	10	0.0057
8	4	5	50	1.0	10	0.0011
9	5	6	50	1.0	10	0.0011

تقييم مُعَوّلية نظام للقدرة الكهربائية باستخدام تحليل شجرة الأخطاء

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ملخص البحث. كنظام كبير ومعقد، قد يخضع نظام القدرة الكهربائية لعدد من الاضطرابات التي يمكن أن تسبب فشلًا كليًا أو جزئيًا في تشغيله. تقتضي هذه الفرضية إدراج تقييم المُعَوّلية في عملية التخطيط لنظام للقدرة. من بين العديد من أساليب تقييم المُعَوّلية المتاحة، يُعتبَر تحليل شجرة الأخطاء بالتأكيد أسلوبًا مناسبًا للغاية ، لا سيما للتعامل مع التعقيد المتأصل في نظم القدرة. تطبق ورقة البحث هذه تحليل شجرة الأخطاء في تقييم جزئي لمُعَوّلية نظام قدرة معين يعرف باسم نظام اختبار روي بيلنتون (ن خ رب). الدراسة المقترحة تستهدف حساب مُعَوّلية توصيل القدرة إلى نقطة الحمل الأكبر في النظام المذكور. تشمل الخطوات التي تم تنفيذها في التحليل إنشاء شجرة الأخطاء، والتحليل الوصفي الكيفي من خلال إنشاء المقاطع الأصغرية، والتحليل الكمي عن طريق حساب المُعَوّلية باستخدام بيانات احتمال الفشل لكل عنصر من عناصر النظام. تتضمن النتائج المقدمة في هذه الدراسة احتمالات الفشل لكل مقطع من المقاطع الأصغرية، واحتمال فشل توصيل الطاقة لنقطة الحمل الأكبر والمُعَوّلية المناظرة، فضلاً عن معايير الأهمية للعناصر المختلفة. ينتج التحليل مجموعة مقاطع تتألف من عشرين مقطعا أصغريا لتوصيل القدرة إلى نقطة الحمل الأكبر في نظام ن خ رب، مع قيمة رقمية مقابلة تبلغ 0.999597 للمُعَوّلية الجزئية للنظام محسوبة عند القيم المفترضة لمعوليات العناصر. تشير تحليلات الأهمية باستخدام أهمية فاصيل وفيزلي وقيمة تحقيق المخاطر وقيمة تقليل المخاطر إلى أن الحافلة الثالثة هي أهم عنصر في نظام توصيل القدرة إلى نقطة الحمل الأكبر، بينما يعد خط النقل الخامس هو أقل مكونات النظام أهمية.